

Radiative transitions of charmonium states in a constituent quark model

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We study the electromagnetic (EM) transitions of the nS , nP ($n \leq 3$), and nD ($n \leq 2$) charmonium states with a constituent quark model. We obtain a reasonable description of the EM transitions of the well-established charmonium states J/ψ , $\psi(2S)$, $\chi_{cJ}(1P)$, $h_c(1P)$ and $\psi(3770)$. We find that the M2 transitions give notable corrections to some E1 dominant processes by interfering with the E1 transitions. Our predictions of EM decay properties for the higher charmonium states are also presented and compared with other model predictions. In particular, we discuss the EM decay properties of some “XYZ” states, such as $X(3823)$, $X(3872)$, $X(3915)$, $X(3940)$ and $X(4350)$ as conventional charmonium states. Assuming $X(3872)$ as the $\chi_{c1}(2P)$ state, our predicted ratio $\Gamma[X(3872) \rightarrow \psi(2S)\gamma]/\Gamma[X(3872) \rightarrow J/\psi\gamma] \simeq 4.0$ is consistent with BaBar’s measurement.

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I. INTRODUCTION

During the past a few years, great progress has been made in the observation of the charmonia [1–3]. From the review of the Particle Data Group (PDG) [4], one can see that a fairly abundant charmonium spectroscopy has been established, and many new charmonium-like “XYZ” states above open-charm thresholds have been discovered from experiments. The observations of these new states not only deepen our understanding of the charmonium physics, but also bring us many mysteries in this field to be uncovered [2, 5]. If these newly observed “XYZ” states are assigned as conventional charmonium states, some aspects, such as measured mass and decay properties are not consistent with the predictions. Thus, on one hand we should test the validity of the previous theoretical models in the descriptions of the new states, and at the same time develop new approaches to study these new states. On the other hand, one should consider these new charmonium-like “XYZ” states as exotic states and attempt to establish a new hadron spectroscopy [6].

Stimulated by the extensive progress made in the observation of the charmonia, in this work we study the electromagnetic (EM) transitions of charmonium in a constituent quark model. As we know, the EM decays of a hadron are sensitive to its inner structure. The study of the EM decays not only is crucial for us to determine the quantum numbers of the newly observed charmonium states, but also provides very useful references for our search for the missing charmonium states in experiments. To deal with the EM decays, beside the widely used potential models [7–11], some other models, such as lattice QCD [12–15], QCD sum rules [16–18], effective Lagrangian approach [19, 20], nonrelativistic effective field theories of QCD [21–23], relativistic quark model [24], relativistic Salpeter method [25], light front quark model [26], and Coulomb gauge approach [27] have been employed in theory. Although some comparable predictions from different models have been obtained, strong model dependencies still exist.

The constituent quark model used in present work has been well developed and widely applied to meson photoproduction reactions [28–40]. Its recent extension to describe the process of πN and KN scattering [41–44] and investigate the strong decays of baryons [45–47] and heavy-light mesons [48–51] also turns out to be successful and inspiring. In this model, to describe the EM transitions of a composite system a special EM operator is adopted, in which the effects of binding potential is included. Furthermore, the possible higher EM multipole contributions to a EM transition process can be included naturally. We expect that our descriptions of the EM transitions of the heavy quarkonium are more successful than those of strong decays of heavy-light mesons or baryons. The reasons are that (i) there is no light quark in the heavy quarkonium, thus, the relativistic effects from the constituent quark are strongly suppressed; (ii) the quark-photon EM coupling used in the EM transitions is well-defined and model independent, however, the nonperturbative quark-meson strong couplings used in the strong decay processes are still effective interactions; (iii) the electric transitions are independent on the heavy constituent quark mass, however, the strong decay processes have some dependencies on the constituent quark mass; (iv) it is natural to consider the emitted photon as a pointlike particle in the EM transitions, however, it is only an approximation to consider the emitted light pseudoscalar meson as a pointlike particle in the strong decay processes.

The paper is organized as follows. In Sec. II, a brief review of charmonium spectroscopy in the constituent quark model is given. In Sec. III, an introduction of EM transitions described in the constituent model is given. The numerical results are presented and discussed in Sec. IV. Finally, a summary is given in Sec. V.

II. CHARMONIUM SPECTROSCOPY

For a quarkonium $Q\bar{Q}$ state, its flavor wave function should have a determined C -parity. For a $C = +1$ state, its flavor

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wave function is

$$\Phi_S = \frac{1}{\sqrt{2}}(Q\bar{Q} + \bar{Q}Q), \quad (1)$$

while for a $C = -1$ state its flavor function is

$$\Phi_A = \frac{1}{\sqrt{2}}(Q\bar{Q} - \bar{Q}Q). \quad (2)$$

The usual spin wave functions are adopted. For the spin $S = 0$ state, it is

$$\chi_0^0 = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \quad (3)$$

and for the spin $S = 1$ states, the wave functions are

$$\chi_1^1 = \uparrow\uparrow, \chi_{-1}^1 = \downarrow\downarrow, \chi_0^1 = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow). \quad (4)$$

In this work, the space wave function of a charmonium state is adopted by the nonrelativistic harmonic oscillator wave function, i.e., $\psi_{Lm}^n = R_{nL}Y_{Lm}$. Where n is the radial quantum number, L is the quantum number of relative orbital angular momentum between quark and antiquark, and m is the quantum number of the third component of L . The detail of the space wave functions can be found in our previous work [48].

The total wave function of a $Q\bar{Q}$ system is a product of the spin-, flavor-, spatial-, and color-wave functions, which should be antisymmetric under the exchange of the two quarks for the constraint of the generalized Pauli principle. The color wave function is always symmetric, thus, the product of the spin-, flavor-, and spatial-wave functions should be antisymmetric. The spectroscopy of some S -, P -, and D -wave charmonium states classified in the constituent quark model has been listed in Tabs. I.

III. EM TRANSITIONS IN THE QUARK MODEL

In this section, we give an introduction of the model used in the calculations. The quark-photon EM coupling at the tree level is described by

$$H_e = - \sum_j e_j \bar{\psi}_j \gamma_\mu^j A^\mu(\mathbf{k}, \mathbf{r}) \psi_j, \quad (5)$$

where ψ_j stands for the j -th quark field in a hadron. The photon has three momentum \mathbf{k} , and the constituent quark ψ_j carries a charge e_j . Replacing the spinor ψ_j by ψ_j^\dagger so that the γ matrices are replaced by the matrix α , the EM transition matrix elements for a radiative decay process can be written as [29]

$$\mathcal{M} = \langle f | \sum_j e_j \alpha_j \cdot \epsilon e^{-i\mathbf{k}\cdot\mathbf{r}_j} | i \rangle, \quad (6)$$

where $|i\rangle$ and $|f\rangle$ stand for the initial and final hadron states, respectively, and ϵ is the polarization vector of the photon.

TABLE I: Charmonium states classified in the constituent quark model. The Clebsch-Gordan series for the spin and angular-momentum addition of the total wave function $|n^{2S+1}L_J\rangle = \sum_{m+S_z=J_z} \langle Lm, S S_z | J J_z \rangle \psi_{Lm}^n \chi_{S_z}^S \Phi$ has been omitted, where $\Phi_{A,S}$ is the flavor wave function. M_{exp} stands for the experimental masses (MeV) from observations, which are taken from the PDG [4]. While M_{NR} , M_{GI} and M_{SNR} stand for the predicted masses with NR, GI and SNR potential models [9, 10].

$n^{2S+1}L_J$	name	J^{PC}	M_{exp}	$M_{\text{NR/GI/SNR}}$	Wave function
1^3S_1	J/ψ	1^{--}	3097	3090/3098/3097	$\psi_{00}^0 \chi_{S_z}^1 \Phi_A$
1^1S_0	$\eta_c(1S)$	0^{-+}	2984	2982/2975/2979	$\psi_{00}^0 \chi^0 \Phi_S$
2^3S_1	$\psi(2S)$	1^{--}	3686	3672/3676/3673	$\psi_{00}^1 \chi_{S_z}^1 \Phi_A$
2^1S_0	$\eta_c(2S)$	0^{-+}	3639	3630/3623/3623	$\psi_{00}^1 \chi^0 \Phi_S$
3^3S_1	$\psi(3S)$	1^{--}	4040	4072/4100/4022	$\psi_{00}^2 \chi_{S_z}^1 \Phi_A$
3^1S_0	$\eta_c(3S)$	0^{-+}	3940?	4043/4064/3991	$\psi_{00}^2 \chi^0 \Phi_S$
1^3P_2	$\chi_{c2}(1P)$	2^{++}	3556	3556/3550/3554	$\psi_{1m}^0 \chi_{S_z}^1 \Phi_S$
1^3P_1	$\chi_{c1}(1P)$	1^{++}	3511	3505/3510/3510	$\psi_{1m}^0 \chi_{S_z}^1 \Phi_S$
1^3P_0	$\chi_{c0}(1P)$	0^{++}	3415	3424/3445/3433	$\psi_{1m}^0 \chi_{S_z}^1 \Phi_S$
1^1P_1	$h_c(1P)$	1^{+-}	3525	3516/3517/3519	$\psi_{1m}^0 \chi^0 \Phi_A$
2^3P_2	$\chi_{c2}(2P)$	2^{++}	3927	3972/3979/3937	$\psi_{1m}^1 \chi_{S_z}^1 \Phi_S$
2^3P_1	$\chi_{c1}(2P)$	1^{++}	3872?	3925/3953/3901	$\psi_{1m}^1 \chi_{S_z}^1 \Phi_S$
2^3P_0	$\chi_{c0}(2P)$	0^{++}	3918?	3852/3916/3842	$\psi_{1m}^1 \chi_{S_z}^1 \Phi_S$
2^1P_1	$h_c(2P)$	1^{+-}		3934/3956/3908	$\psi_{1m}^1 \chi^0 \Phi_A$
3^3P_2	$\chi_{c2}(3P)$	2^{++}	4350?	4317/4337/4208	$\psi_{1m}^2 \chi_{S_z}^1 \Phi_S$
3^3P_1	$\chi_{c1}(3P)$	1^{++}		4271/4317/4178	$\psi_{1m}^2 \chi_{S_z}^1 \Phi_S$
3^3P_0	$\chi_{c0}(3P)$	0^{++}		4202/4292/4131	$\psi_{1m}^2 \chi_{S_z}^1 \Phi_S$
3^1P_1	$h_c(3P)$	1^{+-}		4279/4318/4184	$\psi_{1m}^2 \chi^0 \Phi_A$
1^3D_3	$\psi_3(1D)$	3^{--}		3806/3849/3799	$\psi_{2m}^0 \chi_{S_z}^1 \Phi_A$
1^3D_2	$\psi_2(1D)$	2^{--}	3823	3800/3838/3798	$\psi_{2m}^0 \chi_{S_z}^1 \Phi_A$
1^3D_1	$\psi_1(1D)$	1^{--}	3778	3785/3819/3787	$\psi_{2m}^0 \chi_{S_z}^1 \Phi_A$
1^1D_2	$\eta_{c2}(1D)$	2^{-+}		3799/3837/3796	$\psi_{2m}^0 \chi^0 \Phi_S$
2^3D_3	$\psi_3(2D)$	3^{--}		4167/4217/4103	$\psi_{2m}^1 \chi_{S_z}^1 \Phi_A$
2^3D_2	$\psi_2(2D)$	2^{--}		4158/4208/4100	$\psi_{2m}^1 \chi_{S_z}^1 \Phi_A$
2^3D_1	$\psi_1(2D)$	1^{--}	4191	4142/4194/4089	$\psi_{2m}^1 \chi_{S_z}^1 \Phi_A$
2^1D_2	$\eta_{c2}(2D)$	2^{-+}		4158/4208/4099	$\psi_{2m}^1 \chi^0 \Phi_S$

For a composite system, the relativistic Hamiltonian is taken to be

$$\hat{H} = \sum_j (\alpha_j \cdot \mathbf{p}_j + \beta_j m_j) + \sum_{i,j} V(\mathbf{r}_i - \mathbf{r}_j). \quad (7)$$

By using the identity [29, 52]

$$\alpha_j \equiv i[\hat{H}, \mathbf{r}_j], \quad (8)$$

we have

$$\begin{aligned} \mathcal{M} &= i \left\langle f \left| [\hat{H}, \sum_j e_j \mathbf{r}_j \cdot \epsilon e^{-i\mathbf{k}\cdot\mathbf{r}_j}] \right| i \right\rangle \\ &+ i \left\langle f \left| \sum_j e_j \mathbf{r}_j \cdot \epsilon \alpha_j \cdot \mathbf{k} e^{-i\mathbf{k}\cdot\mathbf{r}_j} \right| i \right\rangle \\ &= -i(E_i - E_f - \omega_\gamma) \langle f | g_e | i \rangle - i\omega_\gamma \langle f | h_e | i \rangle, \end{aligned} \quad (9)$$

with

$$h_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} (1 - \alpha_j \cdot \hat{\mathbf{k}}) e^{-i\mathbf{k} \cdot \mathbf{r}_j}, \quad (10)$$

and

$$g_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{-i\mathbf{k} \cdot \mathbf{r}_j}. \quad (11)$$

In Eq. (9), E_i , E_f and ω_γ stand for the energies of the initial hadron state, final hadron state and emitted photon, respectively. In the initial-hadron-rest system, we have

$$E_i = E_f + \omega_\gamma. \quad (12)$$

Thus, we obtain

$$\mathcal{M} = -i\omega_\gamma \langle f | h_e | i \rangle. \quad (13)$$

In this model, the wave function of a hadron is adopted by the nonrelativistic (NR) harmonic oscillator wave function, i.e., $\psi_{Lm}^n = R_{nL}(r) Y_{Lm}(\Omega)$. To match the NR wave functions of hadrons, we should adopt the NR form of Eq. (10) in the calculations. Following the procedures used in [29, 52], we obtain the NR expansion of h_e in Eq.(10), which is given by

$$h_e \simeq \sum_j \left[e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} - \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{k}}) \right] e^{-i\mathbf{k} \cdot \mathbf{r}_j}. \quad (14)$$

It is interesting to find that the first and second terms in Eq.(14) are responsible for the electric and magnetic transitions, respectively. The second term in Eq.(14) is the same as that used in Ref. [7], while the first term in Eq.(14) differs from $(1/m_j) \mathbf{p}_j \cdot \boldsymbol{\epsilon}$ used in Ref. [7] for the effects of the binding potential is included in the transition operator.

Finally we can relate the standard helicity amplitude \mathcal{A} of the EM decay process to the amplitude \mathcal{M} in Eq.(13) by the relation

$$\mathcal{A} = -i \sqrt{\frac{\omega_\gamma}{2}} \langle f | h_e | i \rangle. \quad (15)$$

It is easily found that the helicity amplitudes for the electric and magnetic transitions are

$$\mathcal{A}^E = -i \sqrt{\frac{\omega_\gamma}{2}} \langle f | \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} e^{-i\mathbf{k} \cdot \mathbf{r}_j} | i \rangle, \quad (16)$$

$$\mathcal{A}^M = +i \sqrt{\frac{\omega_\gamma}{2}} \langle f | \sum_j \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{k}}) e^{-i\mathbf{k} \cdot \mathbf{r}_j} | i \rangle. \quad (17)$$

In the initial-hadron-rest system for the radiative decay process, the momentum of the initial hadron is $\mathbf{P}_i = \mathbf{0}$, and that of the final hadron state is $\mathbf{P}_f = -\mathbf{k}$. Without losing generals, we select the photon momentum along the z axial ($\mathbf{k} = k\hat{\mathbf{z}}$), and take the the polarization vector of the photon with the right-hand form, i.e., $\boldsymbol{\epsilon} = -\frac{1}{\sqrt{2}}(1, i, 0)$, in our calculations. To easily work out the EM transition matrix elements, we use the multipole expansion of the plane wave

$$e^{-i\mathbf{k} \cdot \mathbf{r}_j} = e^{-ikz_j} = \sum_l \sqrt{4\pi(2l+1)} (-i)^l j_l(kr_j) Y_{l0}(\Omega), \quad (18)$$

where $j_l(x)$ is the Bessel function. Then, we obtain the matrix element for the electric multipole transitions with angular momentum l (El transitions):

$$\begin{aligned} \mathcal{A}^{El} = & \sqrt{\frac{\omega_\gamma}{2}} \langle f | \sum_j (-i)^l B_l e_j r_j j_{l+1}(kr_j) Y_{l1}(\Omega) | i \rangle \\ & + \sqrt{\frac{\omega_\gamma}{2}} \langle f | \sum_j (-i)^l B_l e_j r_j j_{l-1}(kr_j) Y_{l1}(\Omega) | i \rangle, \end{aligned} \quad (19)$$

where $B_l \equiv \sqrt{\frac{2\pi l(l+1)}{2l+1}}$. We also obtain the matrix element for the magnetic multipole transitions with angular momentum l (Ml transitions):

$$\mathcal{A}^{Ml} = \sqrt{\frac{\omega_\gamma}{2}} \langle f | \sum_j (-i)^l C_l \frac{e_j \sigma_j}{2m_j} j_{l-1}(kr_j) Y_{l-1,0}(\Omega) | i \rangle \quad (20)$$

where $C_l \equiv i \sqrt{8\pi(2l-1)}$, and $\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y)$ is the spin shift operator. Obviously, the El transitions satisfy the parity selection rule: $\pi_i \pi_f = (-1)^l$; while the Ml transitions satisfy the parity selection rule: $\pi_i \pi_f = (-1)^{l+1}$, where π_i and π_f stand for the parities of the initial and final hadron states, respectively.

Finally, using the parity selection rules, one can express the EM helicity amplitude \mathcal{A} with the matrix elements of EM multipole transitions in a unified form:

$$\mathcal{A} = \sum_l \left\{ \frac{1 + (-1)^{\pi_i \pi_f + l}}{2} \mathcal{A}^{El} + \frac{1 - (-1)^{\pi_i \pi_f + l}}{2} \mathcal{A}^{Ml} \right\}, \quad (21)$$

which is consistent the standard multipole expansion in Ref. [53]. Combining the parity selection rules, we easily know the possible EM multipole contributions to a EM transition considered in present work, which are listed in Tab. II.

TABLE II: Possible EM multipole contributions to a EM transition between two charmonium states.

process	multipole contribution
$n^3 S_1 \longleftrightarrow m^1 S_0$	M1
$n^3 P_J \longleftrightarrow m^3 S_1$	E1, M2
$n^1 P_1 \longleftrightarrow m^1 S_0$	E1
$n^3 D_J \longleftrightarrow m^3 P_J$	E1, E3, M2, M4
$n^1 D_1 \longleftrightarrow m^1 P_1$	E1, E3
$n^3 P_J \longleftrightarrow m^1 P_1$	M1, M3

With these helicity amplitudes \mathcal{A} worked out according to Eq.(21), one obtains the partial decay widths of the EM transitions by

$$\Gamma = \frac{|\mathbf{k}|^2}{\pi} \frac{1}{2J_i + 1} \frac{M_f}{M_i} \sum_{J_{fz}, J_{iz}} |\mathcal{A}_{J_{fz}, J_{iz}}|^2, \quad (22)$$

where J_i is the total angular momenta of the initial mesons, J_{fz} and J_{iz} are the components of the total angular momentum

along the z axis of initial and final mesons, respectively. In the calculation, the standard parameters of the quark model are adopted, no free parameters to be fitted. For the oscillator parameter in the harmonic oscillator wave function we use $\alpha = 400$ MeV, and for the constituent c quark mass we adopt $m_c = 1500$ MeV. For the fine-structure constant, we use $\alpha_{\text{EM}} \equiv e^2/(4\pi) = 1/137$. For the well-established charmonium states, their masses are adopted the experimental average values from the PDG [4]. While for the missing charmonium states, their masses are adopted from the theoretical predictions.

IV. RESULTS AND DISCUSSIONS

A. $J/\psi \rightarrow \eta_c(1S)\gamma$

The $J/\psi \rightarrow \eta_c(1S)\gamma$ is a typical M1 transition. According to our calculation, we find that the partial decay width of this process is proportional to ω_γ^3/m_c^2 . We note that the constituent quark mass $m_c \gg \omega_\gamma$, thus, the transition rate of $J/\psi \rightarrow \eta_c(1S)\gamma$ is strongly suppressed by the factor ω_γ^2/m_c^2 .

About the transition $J/\psi \rightarrow \eta_c(1S)\gamma$, discrepancies still exist between the theoretical predictions and experimental measurements. For example, the calculations from NR potential model [9] and Coulomb gauge approach [27] give a large width $\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma] \simeq 2.9$ keV, which is about a factor of 2 larger than the world average experimental value $\Gamma(J/\psi \rightarrow \eta_c\gamma) \simeq 1.58 \pm 0.37$ keV from the PDG [4]. In this work, our predicted partial decay width for the $J/\psi \rightarrow \eta_c(1S)\gamma$ transition is

$$\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma] \simeq 1.25 \text{ keV}, \quad (23)$$

which is close to the world average data [4], and also in agreement with the values 1.05 keV and (1.5 ± 1.0) keV predicted in the frameworks of relativistic quark model [24] and NR effective field theories of QCD [21, 23], respectively. However, the recent measurements at KEDR give a large partial decay width $\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma] \simeq 2.98 \pm 0.18^{+0.15}_{-0.33}$ keV [54], which is consistent with the lattice QCD results 2.4 – 2.9 keV [12–15], the prediction of Coulomb gauge approach [27], and that of NR and GI potential models [9] (see Tab. III). Finally it should be mentioned that Li and Zhao studied the intermediate hadronic meson loop contributions to the $J/\psi \rightarrow \eta_c(1S)\gamma$ process, which might provide explicit corrections to the M1 transition as well [55, 56]. We hope more efforts on the experimental side will be devoted to clarify the disagreement among various experiments.

B. Radiative transitions of $2S$ states

1. $\psi(2S)$

The $\psi(2S)$ resonance, i.e., $\psi(3686)$, can decay into $\chi_{cJ}(1P)\gamma$ and $\eta_c(1S, 2S)\gamma$ channels, which have been observed in experiments. The $\psi(2S) \rightarrow \chi_{cJ}(1P)\gamma$ decay pro-

cesses are governed by the E1 transitions. While the $\psi(2S) \rightarrow \eta_c(1S, 2S)\gamma$ are typical M1 transitions.

Our predicted partial decay widths for the $\psi(2S) \rightarrow \chi_{cJ}(1P)\gamma$ processes are

$$\Gamma[\psi(2S) \rightarrow \chi_{c0}(1P)\gamma] \simeq 26 \text{ keV}, \quad (24)$$

$$\Gamma[\psi(2S) \rightarrow \chi_{c1}(1P)\gamma] \simeq 22 \text{ keV}, \quad (25)$$

$$\Gamma[\psi(2S) \rightarrow \chi_{c2}(1P)\gamma] \simeq 14 \text{ keV}. \quad (26)$$

From Tab. IV, one can see that our results for the $\psi(2S) \rightarrow \chi_{c0}(1P)\gamma, \chi_{c1}(1P)\gamma$ processes are in good agreement with the experimental data, and the predictions of the GI potential model [9] and relativistic quark model [24]. However, our prediction of the partial decay width $\Gamma[\psi(2S) \rightarrow \chi_{c2}(1P)\gamma] \simeq 14$ keV is about a factor of 1.8 narrower than the world average data [4]. It should be mentioned that according to the study in Ref. [57], the higher term corrections from the coupled-channel effects of intermediate charmed mesons to the $\psi(2S) \rightarrow \chi_{cJ}(1P)\gamma$ processes are negligibly small.

Furthermore, it is interesting to find that our predicted partial width ratios

$$\frac{\Gamma[\psi(2S) \rightarrow \chi_{c2}(1P)\gamma]}{\Gamma[\psi(2S) \rightarrow \chi_{c0}(1P)\gamma]} \simeq 0.54, \quad (27)$$

$$\frac{\Gamma[\psi(2S) \rightarrow \chi_{c1}(1P)\gamma]}{\Gamma[\psi(2S) \rightarrow \chi_{c0}(1P)\gamma]} \simeq 0.85, \quad (28)$$

are in good agreement with the corresponding values 0.60 and 0.86 predicted with NR potential models [9], although their partial widths are larger than ours. We hope these ratios can be measured in future experiments.

Finally, it should be mentioned that the M2 transitions can affect the translations of $\psi(2S) \rightarrow \chi_{cJ}(1P)\gamma$ slightly by interfering with the E1 transitions. Including the effects of M2 transitions, we find that the descriptions of the transitions of $\psi(2S) \rightarrow \chi_{c0}(1P)\gamma, \chi_{c1}(1P)\gamma$ become better compared with the data (see Tab. III).

For the typical M1 transitions $\psi(2S) \rightarrow \eta_c(1S, 2S)\gamma$, our predicted partial decay widths are

$$\Gamma[\psi(2S) \rightarrow \eta_c(1S)\gamma] \simeq 2.41 \text{ keV}, \quad (29)$$

$$\Gamma[\psi(2S) \rightarrow \eta_c(2S)\gamma] \simeq 0.10 \text{ keV}. \quad (30)$$

From Tab. III, we see that our results are close to the average data from the PDG [4] and the predictions of relativistic quark model [24]. The prediction of $\Gamma[\psi(2S) \rightarrow \eta_c(1S)\gamma] \simeq 0.4(8)$ keV from Lattice QCD is notably smaller than our result [13]. However, the partial decay width of $\Gamma[\psi(2S) \rightarrow \eta_c(1S)\gamma]$ is notably overestimated in the framework of GI and NR potential models [9]. It should be emphasized that the intermediate hadronic meson loop might provide explicit corrections to the $\psi(2S) \rightarrow \eta_c(1S, 2S)\gamma$ process [55, 56]. More studies of the M1 transitions $\psi(2S) \rightarrow \eta_c(1S, 2S)\gamma$ are needed in theory.

2. $\eta_c(2S)$

The $\eta_c(2S)$ resonance can decay into $h_c(1P)\gamma$ and $J/\psi\gamma$ by the E1 and M1 transitions, respectively. Our predicted partial

decay width for the $\eta_c(2S) \rightarrow h_c(1P)\gamma$ process is

$$\Gamma[\eta_c(2S) \rightarrow h_c(1P)\gamma] \simeq 18 \text{ keV}. \quad (31)$$

From Tab. IV, it is seen that the predictions of the GI and NR potential models [9] and relativistic quark model [24] are about a factor of 2 larger than our result. While our predicted partial decay width

$$\Gamma[\eta_c(2S) \rightarrow J/\psi\gamma] \simeq 1.64 \text{ keV}, \quad (32)$$

is in good agreement with the prediction of relativistic quark model [24] (see Tab. III). The recent calculation from Lattice QCD gives a fairly large width $\Gamma[\eta_c(2S) \rightarrow J/\psi\gamma] = (15.7 \pm 5.7) \text{ keV}$ [58], which is about an order of magnitude larger than our prediction. Furthermore, the calculations from potential models also give a large width $\Gamma[\eta_c(2S) \rightarrow J/\psi\gamma] = (5.6 \sim 7.9) \text{ keV}$ [9]. The radiative transitions $\eta_c(2S) \rightarrow h_c(1P)\gamma, J/\psi\gamma$ are still not measured in experiments. For the very different predictions of these decays in theory, we hope some observations can be carried out in future experiments.

C. Radiative transitions of $1P$ states

1. $\chi_{cJ}(1P)$

For the triplet $1P$ states $\chi_{cJ}(1P)$ ($J = 0, 1, 2$), their main radiative transitions are $\chi_{cJ}(1P) \rightarrow J/\psi\gamma$. In these decay processes, except for the dominant E1 transitions, the M2 transitions are allowed as well. Neglecting the effects of the M2 transitions, we find the partial decay widths of $\Gamma[\chi_{cJ}(1P) \rightarrow J/\psi\gamma]$ are proportional to $(\omega_\gamma^2/\alpha^2)\omega_\gamma$. We note that the photon energy ω_γ is very close to the oscillator parameter α (i.e., $\omega_\gamma \sim \alpha$), thus, one can easily find that $\Gamma[\chi_{cJ}(1P) \rightarrow J/\psi\gamma] \sim \omega_\gamma$. We calculate the partial decay widths $\Gamma[\chi_{cJ}(1P) \rightarrow J/\psi\gamma]$, our results are

$$\Gamma[\chi_{c0}(1P) \rightarrow J/\psi\gamma] \simeq 128 \text{ keV}, \quad (33)$$

$$\Gamma[\chi_{c1}(1P) \rightarrow J/\psi\gamma] \simeq 275 \text{ keV}, \quad (34)$$

$$\Gamma[\chi_{c2}(1P) \rightarrow J/\psi\gamma] \simeq 467 \text{ keV}. \quad (35)$$

From Tab. IV, we find that our predictions of $\Gamma[\chi_{c0,1}(1P) \rightarrow J/\psi\gamma]$ are in good agreement with the world average data from the PDG [4]. While our prediction of $\Gamma[\chi_{c2}(1P) \rightarrow J/\psi\gamma]$ is slightly (a factor of 1.26) larger than the present average data $(371 \pm 34) \text{ keV}$ [4]. It is interesting to find that the E1 transitions of $\chi_{cJ}(1P) \rightarrow J/\psi\gamma$ predicted by us are in good agreement with those of NR potential model [9]. Furthermore, to understand the radiative transition properties of $\chi_{c0,1}(1P)$ some studies were carried out from Lattice QCD as well [12, 14], however, good descriptions are still not obtained for some technical problems.

It should be emphasized that the M2 transitions have obvious corrections to the radiative transitions of $\chi_{cJ}(1P) \rightarrow J/\psi\gamma$ by interfering with the E1 transitions. In the $\chi_{c2}(1P) \rightarrow J/\psi\gamma$ process, the M2 transition has a constructive interference with the E1 transition, while in the $\chi_{c1,0}(1P) \rightarrow J/\psi\gamma$ processes,

the M2 transition has a destructive interference with the E1 transition. The corrections from the M2 transition might reach to (10 ~ 20)% of the partial decay widths (see Tab. IV). Furthermore, the partial width ratios were also obviously affected by the corrections from the M2 transition. For example, considering the M2 contributions, we find that the partial width ratio of

$$\frac{\Gamma[\chi_{c2}(1P) \rightarrow J/\psi\gamma]}{\Gamma[\chi_{c0}(1P) \rightarrow J/\psi\gamma]} \simeq 3.64 \quad (36)$$

is notably larger than the value 2.54 without M2 contributions. This ratio is expected to be tested by more precise measurements in the future.

Recently, the coupled-channel effects of intermediate charmed mesons on the radiative transitions $\chi_{cJ}(1P) \rightarrow J/\psi\gamma$ were also studied with an effective Lagrangian approach in Ref. [57], the results show that the coupled-channel effects on these decay processes are relatively weak.

2. $h_c(1P)$

For the singlet $h_c(1P)$ state, its main radiative transition is $h_c(1P) \rightarrow \eta_c(1S)\gamma$. It is a typical E1 transition. The partial width of $\Gamma[h_c(1P) \rightarrow \eta_c(1S)\gamma]$ is also proportional to $(\omega_\gamma^2/\alpha^2)\omega_\gamma$. Thus, the width of $\Gamma[h_c(1P) \rightarrow \eta_c(1S)\gamma]$ is the same order of magnitude as $\Gamma[\chi_{cJ}(1P) \rightarrow J/\psi\gamma]$. Our predicted partial decay width

$$\Gamma[h_c(1P) \rightarrow \eta_c\gamma] \simeq 587 \text{ keV}, \quad (37)$$

is in good agreement with the predictions from the relativistic quark model [24] and Lattice QCD [12], and also consistent with the data within its large uncertainties [4] (see table IV). The NR and GI potential models and Coulomb gauge approach [27] give a notably narrower width than ours [9]. It should be mentioned that the recent lattice calculation gave a larger width of $\Gamma[h_c(1P) \rightarrow \eta_c\gamma] = 720(70) \text{ keV}$. Similar result was also obtained with a light front quark model [26]. As a whole, there are large discrepancies between different model predictions of $\Gamma[h_c(1P) \rightarrow \eta_c\gamma]$. However, the present world data can not be used to test the various predictions for their large uncertainties. More accurate measurements for the transition $h_c(1P) \rightarrow \eta_c\gamma$ are needed.

Finally, we give an estimation of partial decay width for the M1 transition $h_c(1P) \rightarrow \chi_{c0}(1P)\gamma$. Our result is

$$\Gamma[h_c(1P) \rightarrow \chi_{c0}(1P)\gamma] = 0.39 \text{ keV}. \quad (38)$$

The corresponding branching ratio is

$$\mathcal{B}[h_c(1P) \rightarrow \chi_{c0}(1P)\gamma] \simeq 5.6 \times 10^{-4}. \quad (39)$$

This sizeable transition rate indicates that the M1 transition $h_c(1P) \rightarrow \chi_{c0}(1P)\gamma$ might be observed in forthcoming experiments.

D. Radiative transitions of 1D states

1. $\psi(3770)$

The $\psi(3770)$ resonance is primarily a $\psi_1(1D)$ state with small admixtures of $\psi(2S)$ [1]. It can decay into $\chi_{cJ}(1P)\gamma$. These decay processes are dominated by the E1 transition. Furthermore, the higher order E3, M2 and M4 transitions are allowed as well.

Considering $\psi(3770)$ as a pure $\psi_1(1D)$ state, we calculate the radiative decay widths of $\Gamma[\psi(3770) \rightarrow \chi_{cJ}(1P)\gamma]$. Our results

$$\Gamma[\psi(3770) \rightarrow \chi_{c0}(1P)\gamma] \simeq 218 \text{ keV}, \quad (40)$$

$$\Gamma[\psi(3770) \rightarrow \chi_{c1}(1P)\gamma] \simeq 70 \text{ keV}, \quad (41)$$

$$\Gamma[\psi(3770) \rightarrow \chi_{c2}(1P)\gamma] \simeq 2.6 \text{ keV}, \quad (42)$$

are consistent with the world average data from PDG [4], and also very close to the predictions of GI model [9] (see Tab. VI). It is interesting to find that recently the BESIII Collaboration gave their precise measurement of the partial width of $\Gamma[\psi(3770) \rightarrow \chi_{c1}(1P)\gamma] = 67.5 \pm 10.8 \text{ keV}$ [79], which is in good agreement with our prediction. Furthermore, our predicted partial width ratio

$$\frac{\Gamma[\psi(3770) \rightarrow \chi_{c0}(1P)\gamma]}{\Gamma[\psi(3770) \rightarrow \chi_{c1}(1P)\gamma]} \simeq 3.1, \quad (43)$$

is in good agreement with the experimental data 2.5 ± 0.6 from CLEO Collaboration as well [59].

It should be emphasized that the M2 transitions also have an obvious contribution [about (10 ~ 15)%] to the radiative transitions of $\Gamma[\psi(3770) \rightarrow \chi_{cJ}(1P)\gamma]$ by interfering with the E1 transitions.

Finally, it should be mentioned that the present measurements for the radiative transitions of $\psi(3770) \rightarrow \chi_{cJ}(1P)\gamma$ are not precise enough to determine the small mixing angle between $\psi_1(1D)$ and $\psi(2S)$.

2. $X(3823)$

Recently, $X(3823)$ as a good candidate for $\psi_2(1D)$ was observed by the Belle Collaboration in the $B \rightarrow \chi_{c1}\gamma K$ decay with a statistical significance of 3.8σ [60]. Lately, this state was confirmed by the BESIII Collaboration in the process $e^+e^- \rightarrow \pi^+\pi^-X(3823) \rightarrow \pi^+\pi^-\chi_{c1}\gamma$ with a statistical significance of 6.2σ [61].

Considering $X(3823)$ as the $\psi_2(1D)$ state, we predict the radiative decay widths of $\Gamma[X(3823) \rightarrow \chi_{cJ}(1P)\gamma]$. Our results are

$$\Gamma[X(3823) \rightarrow \chi_{c0}(1P)\gamma] \simeq 1.42 \text{ keV}, \quad (44)$$

$$\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma] \simeq 227 \text{ keV}, \quad (45)$$

$$\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma] \simeq 42 \text{ keV}. \quad (46)$$

Where we can find that the radiative transition rate of $X(3823) \rightarrow \chi_{c0}(1P)\gamma$ is relatively small. The radiative transitions of $X(3823)$ are dominated by the $\chi_{c1}(1P)\gamma$ channel.

This can explain why the $X(3823)$ is firstly observed in the $\chi_{c1}(1P)\gamma$ channel, while not observed in the $\chi_{c0,c2}(1P)\gamma$ channels. It should be mentioned that the M2 transitions could give a $\sim 15\%$ correction to $\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]$ by interfering with the E1 transitions. Furthermore, Our predicted partial width ratio,

$$\frac{\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]}{\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma]} \simeq 19\%, \quad (47)$$

is consistent with the observations $< 42\%$ [61]. Our predictions of the $\Gamma[X(3823) \rightarrow \chi_{c1,c2}(1P)\gamma]$ are compatible with the other theoretical predictions [9, 24, 62, 63].

According to the calculations from various models, the partial width of $\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]$ is not small. Thus, looking for $X(3823)$ in the $\chi_{c2}(1P)\gamma$ channel and an accurate measurement of the ratio $\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]/\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma]$ are crucial to further confirm $X(3823)$ as the $\psi_2(1D)$ state.

3. $\psi_3(1D)$ and $\eta_{c2}(1D)$

Another two 1D-wave states $\psi_3(1D)$ and $\eta_{c2}(1D)$ have not been observed in experiments. According to theoretical predictions, their masses are very similar to that of $\psi_2(1D)$. If $X(3823)$ corresponds to the $\psi_2(1D)$ state indeed, the masses of the $\psi_3(1D)$ and $\eta_{c2}(1D)$ resonances should be around 3.82 GeV.

For the singlet 1D state $\eta_{c2}(1D)$, its main radiative transition is $\eta_{c2}(1D) \rightarrow h_c(1P)\gamma$. This process is governed by the E1 transition, the effects from the E3 transition are negligibly small. Taking the mass of $\eta_{c2}(1D)$ with $M = 3820 \text{ MeV}$, we predict the partial decay width

$$\Gamma[\eta_{c2}(1D) \rightarrow h_c(1P)\gamma] \simeq 261 \text{ keV}. \quad (48)$$

Our result is compatible with that from the relativistic quark model [24] and effective Lagrangian approach [19]. The predictions from the potential models [9, 10] are slightly larger than ours (see Tab. VI).

While for the triplet 1D state $\psi_3(1D)$, its common radiative transitions are $\psi_3(1D) \rightarrow \chi_{cJ}(1P)\gamma$. In these transitions, E1, E3, M2, M4 decays are allowed. Taking the mass of $\psi_3(1D)$ with $M = 3830 \text{ MeV}$, we calculate the partial decay widths $\Gamma[\psi_3(1D) \rightarrow \chi_{cJ}(1P)\gamma]$. Our results are

$$\Gamma[\psi_3(1D) \rightarrow \chi_{c0}(1P)\gamma] \simeq 0.87 \text{ keV}, \quad (49)$$

$$\Gamma[\psi_3(1D) \rightarrow \chi_{c1}(1P)\gamma] \simeq 0.82 \text{ keV}, \quad (50)$$

$$\Gamma[\psi_3(1D) \rightarrow \chi_{c2}(1P)\gamma] \simeq 226 \text{ keV}. \quad (51)$$

The interferences between M2 and E3 transitions are responsible for the tiny partial decay widths of $\Gamma[\psi_3(1D) \rightarrow \chi_{c0,c1}(1P)\gamma]$, while the E1 transitions govern the partial width of $\Gamma[\psi_3(1D) \rightarrow \chi_{c2}(1P)\gamma]$. The magnitude of the partial decay width of $\Gamma[\psi_3(1D) \rightarrow \chi_{c2}(1P)\gamma]$ predicted by us is compatible with that from the potential models [9, 10] and the relativistic quark model [24]. We hope the experimental Collaboration can carry out a search for the missing $\psi_3(1D)$ state in the $\chi_{c2}(1P)\gamma$ channel.

E. Radiative transitions of $2P$ states

1. $\chi_{c2}(2P)$

In the $2P$ charmonium states, only the $\chi_{c2}(2P)$ has been established experimentally. This state was observed by both Belle [64] and BaBar [65] in the two-photon fusion process $\gamma\gamma \rightarrow D\bar{D}$ with a mass $M \simeq 3927$ MeV and a narrow width $\Gamma \simeq 24$ MeV [4]. We analyze its radiative transitions to $\psi(1D)\gamma$, $J/\psi\gamma$ and $\psi(2S)\gamma$. We find the radiative transition rate to $\psi(1D)\gamma$ is very weak. The predicted partial decay widths are

$$\Gamma[\chi_{c2}(2P) \rightarrow \psi(3770)\gamma] \simeq 0.32 \text{ keV}, \quad (52)$$

$$\Gamma[\chi_{c2}(2P) \rightarrow \psi_2(1D)\gamma] \simeq 1.5 \text{ keV}, \quad (53)$$

$$\Gamma[\chi_{c2}(2P) \rightarrow \psi_3(1D)\gamma] \simeq 6.3 \text{ keV}. \quad (54)$$

In the calculation we take the mass of 3830 MeV for $\psi_3(1D)$. Our predictions of the $\Gamma[\chi_{c2}(2P) \rightarrow \psi(3770)\gamma]$ and $\Gamma[\chi_{c2}(2P) \rightarrow \psi_2(1D)\gamma]$ are roughly compatible with the results from the GI potential model [9] (see Tab. IV). However, our prediction of the $\Gamma[\chi_{c2}(2P) \rightarrow \psi_3(1D)\gamma]$ is about an order of magnitude smaller than that from the GI and NR potential models [9] (see Tab. V). Our predictions indicate that it might be a challenge to look for the missing charmonium state $\psi_3(1D)$ via the radiative transition $\chi_{c2}(2P) \rightarrow \psi_3(1D)\gamma$.

The $\chi_{c2}(2P)$ state might have large radiative decay rates into $J/\psi\gamma$ and $\psi(2S)\gamma$. In our calculations, we find

$$\Gamma[\chi_{c2}(2P) \rightarrow J/\psi\gamma] \simeq 34 \text{ keV}, \quad (55)$$

$$\Gamma[\chi_{c2}(2P) \rightarrow \psi(2S)\gamma] \simeq 133 \text{ keV}. \quad (56)$$

It should be mentioned that the M2 transitions could give a $\sim 10 - 15\%$ correction to these radiative decay widths by interfering with the E1 transitions. The branching fractions of these two radiative decay process could reach to

$$\mathcal{B}[\chi_{c2}(2P) \rightarrow J/\psi\gamma] \simeq 1.4 \times 10^{-3} \text{ keV}, \quad (57)$$

$$\mathcal{B}[\chi_{c2}(2P) \rightarrow \psi(2S)\gamma] \simeq 5.5 \times 10^{-3} \text{ keV}. \quad (58)$$

Thus, the radiative transitions $\chi_{c2}(2P) \rightarrow \psi(2S)\gamma, J/\psi\gamma$ are most likely to be observed in forthcoming experiments.

Finally, it should be pointed out that our predicted partial width ratio

$$\frac{\Gamma[\chi_{c2}(2P) \rightarrow \psi(2S)\gamma]}{\Gamma[\chi_{c2}(2P) \rightarrow J/\psi\gamma]} \simeq 4, \quad (59)$$

is in good agreement with that of GI and NR potential model, though their predicted partial widths are about a factor of 2 – 3 larger than our results. Our predicted ratio is slightly larger than the result 2.95 ± 0.5 predicted with an effective Lagrangian approach [19]. To better understand the properties of the $\chi_{c2}(2P)$ state and test the various theoretical predictions, this ratio is suggested to be measured in experiments.

2. $\chi_{c1}(2P)$

The $X(3872)$ resonance has the same quantum numbers as $\chi_{c1}(2P)$ (i.e., $J^{PC} = 1^{++}$) and a similar mass to the predicted

value of $\chi_{c1}(2P)$. However, its exotic properties can not be well understood with a pure $\chi_{c1}(2P)$ state [5, 6]. To understand the nature of $X(3872)$, measurements of the radiative decays of $X(3872)$ have been carried out by the BaBar [66], Belle [67], and LHCb [68] collaborations, respectively. Obvious evidence of $X(3872) \rightarrow J/\psi\gamma$ was observed by these collaborations. Furthermore, the BaBar and LHCb Collaborations also observed evidence of $X(3872) \rightarrow \psi(2S)\gamma$. The branching fraction ratio

$$R_{\psi'\gamma/\psi\gamma}^{\text{exp}} = \frac{\Gamma[X(3872) \rightarrow \psi(2S)\gamma]}{\Gamma[X(3872) \rightarrow J/\psi\gamma]} \simeq 3.4 \pm 1.4, \quad (60)$$

obtained by the BaBar Collaboration [66] is consistent with the recent measurement $R_{\psi'\gamma/\psi\gamma}^{\text{exp}} = 2.46 \pm 0.93$ of LHCb Collaboration [68].

Considering the $X(3872)$ as a pure $\chi_{c1}(2P)$ state, we calculate the radiative decays $X(3872) \rightarrow J/\psi\gamma, \psi(2S)\gamma$. Our predicted radiative decay partial widths are

$$\Gamma[X(3872) \rightarrow J/\psi\gamma] \simeq 14.4 \text{ keV}, \quad (61)$$

$$\Gamma[X(3872) \rightarrow \psi(2S)\gamma] \simeq 57.1 \text{ keV}. \quad (62)$$

It should be mentioned that the M2 transitions could give a $\sim 10\%$ correction to the radiative decay widths. Combining these predicted partial widths, we can easily obtain the ratio

$$R_{\psi'\gamma/\psi\gamma}^{\text{th}} = \frac{\Gamma[X(3872) \rightarrow \psi(2S)\gamma]}{\Gamma[X(3872) \rightarrow J/\psi\gamma]} \simeq 4.0, \quad (63)$$

which is consistent with the BaBar's measurement [66], and close to the upper limit of the observations from LHCb [68]. The ratio predicted by us is also in agreement with the result $R_{\psi'\gamma/\psi\gamma}^{\text{th}} \simeq 4.4$ from a relativistic Salpeter method [10, 25]. It should be mentioned that Barnes and Godfrey gave a ratio $R_{\psi'\gamma/\psi\gamma}^{\text{th}} \simeq 6$ [8], which is obviously larger than ours although their predicted partial widths are close to ours.

Furthermore, considering $X(3872)$ as a pure $\chi_{c1}(2P)$ state, we calculate the radiative decays into the D -wave states $\psi(3770)$ and $\psi_2(1D)$ (i.e., $X(3823)$):

$$\Gamma[X(3872) \rightarrow \psi(3770)\gamma] \simeq 1.8 \text{ keV}, \quad (64)$$

$$\Gamma[X(3872) \rightarrow \psi_2(1D)\gamma] \simeq 0.77 \text{ keV}. \quad (65)$$

These partial widths are much smaller than the partial widths into the S -wave states. Combining the measured width of $X(3872)$ (i.e., $\Gamma < 1.2$ MeV) from the PDG [4], we estimate the branching ratios

$$\mathcal{B}[X(3872) \rightarrow \psi(3770)\gamma] > 1.9 \times 10^{-3}, \quad (66)$$

$$\mathcal{B}[X(3872) \rightarrow \psi_2(1D)\gamma] > 6.4 \times 10^{-4}. \quad (67)$$

The sizeable decay rates indicate that the decay modes $\psi(3770)\gamma$ and $\psi_2(1D)\gamma$ might be observed in experiments if $X(3872)$ corresponds to the $\chi_{c1}(2P)$ state indeed.

As a whole, from the view of branching fraction ratio $R_{\psi'\gamma/\psi\gamma}$, our result supports $X(3872)$ as a candidate of $\chi_{c1}(2P)$, which is in agreement with the predictions in Refs. [10, 25]. More observations in the $\psi(3770)\gamma$ and $\psi_2(1D)\gamma$ channels are useful to understand the nature of $X(3872)$.

3. $\chi_{c0}(2P)$

The $\chi_{c0}(2P)$ state is still not well-established, although $X(3915)$ was recommended as the $\chi_{c0}(2P)$ state in Ref. [69], and also assigned as the $\chi_{c0}(2P)$ state by the PDG [4] recently. Assigning the $X(3915)$ as the $\chi_{c0}(2P)$ state will face several serious problems [70, 71]. For example, the mass splitting between $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$, the production rates, and limits on the $\omega J/\psi$ and $D\bar{D}$ branching fractions of $X(3915)$ can not be well explained. Guo and Meissner refitted the BaBar and Belle data of $\gamma\gamma \rightarrow D\bar{D}$ separately, their analysis indicates that the broad bump in the invariant mass spectrum could be assigned as the $\chi_{c0}(2P)$ state [70]. Its average mass and width are $M = (3837.6 \pm 11.5) \text{ MeV}$ and $\Gamma = (221 \pm 19) \text{ MeV}$, respectively [70]. The extracted mass of $\chi_{c0}(2P)$ is consistent with the predictions in the screened potential model [10] and relativistic quark model [24]. Recently, Zhou *et al.* carried out a combined amplitude analysis of the $\gamma\gamma \rightarrow D\bar{D}, \omega J/\psi$ data [72]. They demonstrated that $X(3915)$ and $X(3930)$ can be regarded as the same state with $J^{PC} = 2^{++}$ (i.e., $\chi_{c2}(2P)$). To establish the $\chi_{c0}(2P)$ state and clarify the controversial situation of $X(3915)$, a study of the radiative transitions of $\chi_{c0}(2P)$ in both theory and experiment is necessary.

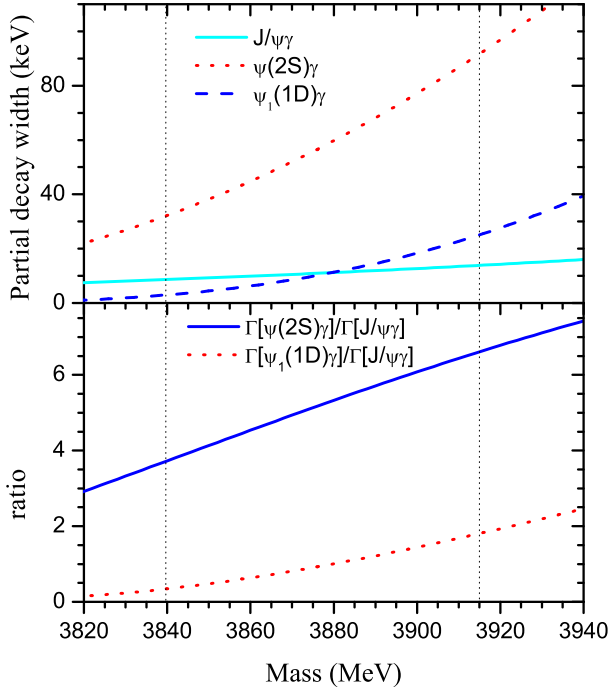


FIG. 1: (Color online) The partial widths of the radiative transitions of $\chi_{c0}(2P)$, and their ratios as a function of the mass in the range of (3820 ~ 3940) GeV.

The $\chi_{c0}(2P)$ state can decay via the radiative transitions $\chi_{c0}(2P) \rightarrow \psi(3770)\gamma, \psi(2S)\gamma, J/\psi\gamma$. For we do not know the accurate mass of $\chi_{c0}(2P)$, we calculate the properties of

these radiative transitions as a function of the mass in the region (3820 ~ 3940) GeV. Our results are shown in Fig. 1. From the figure, one can find that the radiative transitions of $\chi_{c0}(2P) \rightarrow \psi(3770)\gamma, \psi(2S)\gamma$ are sensitive to the mass of $\chi_{c0}(2P)$.

If $\chi_{c0}(2P)$ has a low mass of $M \simeq 3840 \text{ MeV}$ as suggested by Guo and Meissner [70], $\psi(2S)\gamma$ and $J/\psi\gamma$ are the important radiative decay modes. The predicted partial decay widths are

$$\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma] \simeq 8.6 \text{ keV}, \quad (68)$$

$$\Gamma[\chi_{c0}(2P) \rightarrow \psi(2S)\gamma] \simeq 32 \text{ keV}, \quad (69)$$

$$\Gamma[\chi_{c0}(2P) \rightarrow \psi(3770)\gamma] \simeq 2.1 \text{ keV}, \quad (70)$$

and the predicted partial width ratios are

$$\frac{\Gamma[\chi_{c0}(2P) \rightarrow \psi(2S)\gamma]}{\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma]} \simeq 3.7, \quad (71)$$

$$\frac{\Gamma[\chi_{c0}(2P) \rightarrow \psi(3770)\gamma]}{\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma]} \simeq 0.24. \quad (72)$$

On the other hand, if $\chi_{c0}(2P)$ has a high mass of $M \simeq 3915 \text{ MeV}$ as suggested by the PDG [4], the radiative decay mode $\psi(3770)\gamma$ also becomes important. Our predicted partial decay widths are

$$\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma] \simeq 14 \text{ keV}, \quad (73)$$

$$\Gamma[\chi_{c0}(2P) \rightarrow \psi(2S)\gamma] \simeq 92 \text{ keV}, \quad (74)$$

$$\Gamma[\chi_{c0}(2P) \rightarrow \psi(3770)\gamma] \simeq 21 \text{ keV}. \quad (75)$$

While the predicted partial width ratios are

$$\frac{\Gamma[\chi_{c0}(2P) \rightarrow \psi(2S)\gamma]}{\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma]} \simeq 6.6, \quad (76)$$

$$\frac{\Gamma[\chi_{c0}(2P) \rightarrow \psi(3770)\gamma]}{\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma]} \simeq 1.5. \quad (77)$$

According to our calculations, we find that the properties of the radiative transitions of $\chi_{c0}(2P)$ between the low- and high-mass cases are very different. Thus, we suggest our experimental colleagues observe the $\chi_{c0}(2P)$ state in the $\psi(2S)\gamma, J/\psi\gamma, \psi(3770)\gamma$ decay channels and measure these partial width ratios, which might provide us a good chance to clarify the puzzles about the $\chi_{c0}(2P)$ state.

4. $h_c(2P)$

There is no information of $h_c(2P)$ from experiments. According to the model predictions, the mass-splitting between $\chi_{c2}(2P)$ and $h_c(2P)$ is about $M_{\chi_{c2}(2P)} - M_{h_c(2P)} = (30 \pm 10) \text{ MeV}$ [9–11]. Thus, the mass of $h_c(2P)$ is most likely to be $M_{h_c(2P)} \simeq 3900 \text{ MeV}$. The typical radiative decay channels of $h_c(2P)$ are $\eta_c(1S, 2S)\gamma$ and $\eta_{c2}(1D)\gamma$.

Considering the uncertainties of the mass, in Fig. 2 we plot the partial decay widths as a function of mass in the range of $M_{h_c(2P)} = (3900 \pm 20) \text{ MeV}$. It is found that the radiative transition rate of $h_c(2P) \rightarrow \eta_{c2}(1D)\gamma$ is very weak. While $h_c(2P)$ has large transition rates to the $\eta_c(1S)\gamma$ and $\eta_c(2S)\gamma$ channels.

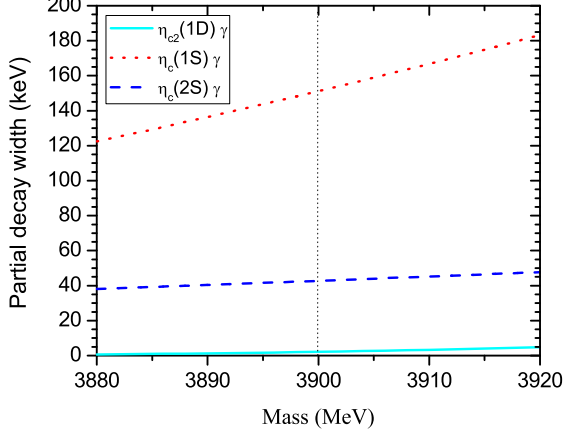


FIG. 2: (Color online) The partial decay widths of $h_c(2P)$ as a function of its mass in the range of $M_{h_c(2P)} = (3900 \pm 20)$ MeV.

The $h_c(2P) \rightarrow \eta_c(1S)\gamma$ transition shows some sensitivities to the mass of $h_c(2P)$. With $M_{h_c(2P)} \simeq (3900 \pm 20)$ MeV, our predicted partial widths are

$$\Gamma(h_c(2P) \rightarrow \eta_c(1S)\gamma) \simeq 151 \pm 30 \text{ keV}, \quad (78)$$

$$\Gamma(h_c(2P) \rightarrow \eta_c(2S)\gamma) \simeq 43 \pm 5 \text{ keV}. \quad (79)$$

The rather sizeable partial widths of $h_c(2P) \rightarrow \eta_c(1S, 2S)\gamma$ are also obtained in the potential model calculations [9–11]. Thus, the radiative transitions $h_c(2P) \rightarrow \eta_c(1S, 2S)\gamma$ are worth to observing in experiments.

Recently, the BESIII Collaboration observed a new neutral charmonium-like particle $Z_c(3900)^0$ in the $e^+e^- \rightarrow \pi^0\pi^0 J/\psi$ process [73], which is likely the isospin partner of $Z_c(3900)^\pm$. It should be mentioned that the possibility of the $Z_c(3900)^0$ as a candidate of $h_c(2P)$ should be considered carefully as well.

F. Radiative transitions of 3S states

1. $\psi(4040)$

The $\psi(4040)$ resonance is commonly identified with the $\psi(3S)$ state [1]. This state can decay into $\chi_{cJ}(1P)\gamma$ and $\chi_{cJ}(2P)\gamma$ via the radiative transitions. We have calculated these precesses. According to our calculations, it is found that the radiative transition rates of $\psi(4040) \rightarrow \chi_{cJ}(1P)\gamma$ are relatively weak. Our predicted partial widths are

$$\Gamma[\psi(4040) \rightarrow \chi_{c0}(1P)\gamma] \simeq 3.1 \text{ keV}, \quad (80)$$

$$\Gamma[\psi(4040) \rightarrow \chi_{c1}(1P)\gamma] \simeq 3.0 \text{ keV}, \quad (81)$$

$$\Gamma[\psi(4040) \rightarrow \chi_{c2}(1P)\gamma] \simeq 2.2 \text{ keV}. \quad (82)$$

The small partial decay widths of $\Gamma[\psi(4040) \rightarrow \chi_{cJ}(1P)\gamma]$ are also predicted with potential models [9, 11]. Recently, the Belle Collaboration observed the processes $e^+e^- \rightarrow \chi_{c1,c2}(1P)\gamma$ [74], they only gave upper limits on branching fractions $\mathcal{B}[\psi(4040) \rightarrow \chi_{c1}(1P)\gamma] = 3.4 \times 10^{-3}$ and

$\mathcal{B}[\psi(4040) \rightarrow \chi_{c2}(1P)\gamma] = 5.5 \times 10^{-3}$. Combining the measured decay width, one obtains the partial decay widths of $\Gamma[\psi(4040) \rightarrow \chi_{c1}\gamma] < 272 \text{ keV}$ and $\Gamma[\psi(4040) \rightarrow \chi_{c2}\gamma] < 440 \text{ keV}$ from experiments. The upper limits of these partial widths from experiments are too large to give any constraints of the theoretical predictions. It might be a challenge to carry out accurate observations of the radiative transitions $\psi(4040) \rightarrow \chi_{cJ}(1P)\gamma$ in experiments for their small decay rates.

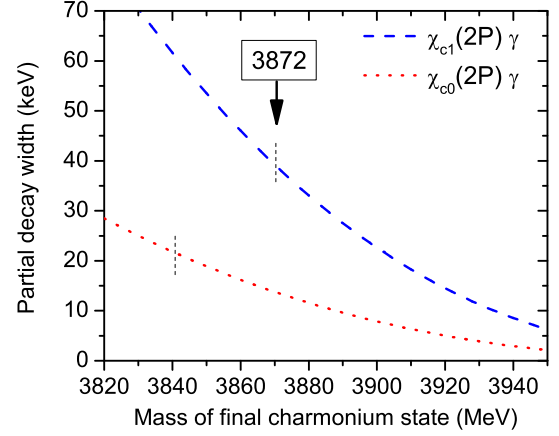


FIG. 3: (Color online) The partial decay widths of the radiative transitions $\psi(4040) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ as functions of the masses of the final states $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$.

It is interesting to find that the radiative transition rates of $\psi(4040) \rightarrow \chi_{cJ}(2P)\gamma$ are not small in theory. The decay rates to the $\chi_{cJ}(2P)\gamma$ channels are about one order of magnitude larger than those of the $\chi_{cJ}(1P)\gamma$ channels. We predict

$$\Gamma[\psi(4040) \rightarrow \chi_{c2}(2P)\gamma] \simeq 19 \text{ keV}. \quad (83)$$

Our result is compatible with the NR potential model prediction [9]. The $\chi_{c2}(2P)\gamma$ decay mode of $\psi(4040)$ is likely to be observed in forthcoming experiments.

The $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states are still not established, thus, we calculate the partial decay widths of the radiative transitions $\psi(4040) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ as functions of the masses of the $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states in a possible region. Our results are shown in Fig. 3. If the $X(3872)$ is the $\chi_{c1}(2P)$ state, our predicted partial decay width of the process $\psi(4040) \rightarrow \chi_{c1}(2P)\gamma$ is

$$\Gamma[\psi(4040) \rightarrow \chi_{c1}(2P)\gamma] \simeq 38 \text{ keV}. \quad (84)$$

While if the mass of $\chi_{c0}(2P)$ is ~ 3840 MeV as suggested in Refs. [10, 70], the predicted partial decay width of the process $\psi(4040) \rightarrow \chi_{c0}(2P)\gamma$ is

$$\Gamma[\psi(4040) \rightarrow \chi_{c0}(2P)\gamma] \simeq 22 \text{ keV}. \quad (85)$$

Thus, if $X(3872)$ is identified as $\chi_{c1}(2P)$ indeed, and $\chi_{c0}(2P)$ has a low mass of ~ 3840 MeV, both $\chi_{c1}(2P)$ and $\chi_{c0}(2P)$ could be produced through the radiative decays of $\psi(4040)$, and established in the $J/\psi\gamma$ and $\psi(2S)\gamma$ final states.

2. $\eta_c(3S)$

The $\eta_c(3S)$ state is not established in experiments. According to model predictions, its mass is about 40–100 MeV lower than that of $\psi(4040)$ [9, 10]. Thus, its mass is most likely to be in the range of 3940–4000 MeV. The $X(3940)$ resonance observed by the Belle Collaboration in $e^+e^- \rightarrow J/\psi X$ [75, 76] is a good candidate of $\eta_c(3S)$ by comparing the observed decay mode, width, and mass with those predicted in theory [1].

Considering $X(3940)$ as the $\eta_c(3S)$ state, we calculate the radiative transitions $\eta_c(3S) \rightarrow h_c(1P)\gamma, h_c(2P)\gamma$. Our predicted partial decay widths

$$\Gamma[\eta_c(3940) \rightarrow h_c(1P)\gamma] \simeq 1.8 \text{ keV}, \quad (86)$$

$$\Gamma[\eta_c(3940) \rightarrow h_c(2P)\gamma] \simeq 1.7 \text{ keV}, \quad (87)$$

are relatively small. Combining the observed width of $X(3940)$, i.e., $\Gamma \simeq 37 \text{ MeV}$, we easily predict the branching fractions

$$\mathcal{B}[\eta_c(3940) \rightarrow h_c(1P)\gamma] \simeq 4.9 \times 10^{-5}, \quad (88)$$

$$\mathcal{B}[\eta_c(3940) \rightarrow h_c(2P)\gamma] \simeq 4.6 \times 10^{-5}. \quad (89)$$

The small branching ratios indicate that if $X(3940)$ is the $\eta_c(3S)$ state, it might be difficult to be observed in the radiative decay channels $h_c(1P)\gamma$ and $h_c(2P)\gamma$. Note that we have taken the mass of $h_c(2P)$ with $M_{h_c(2P)} = 3.9 \text{ GeV}$ in the calculation. Finally, it should be mentioned that our predicted partial widths are notably smaller than those of NR and GI potential models [9] (see Tab. V).

G. Radiative transitions of 2D states

1. $\psi(4160)$

The 1^{--} state $\psi(4160)$ is commonly identified with the 2^3D_1 state. The average experimental mass and width from the PDG are $M = 4191 \pm 5 \text{ MeV}$ and $\Gamma = 70 \pm 10 \text{ MeV}$, respectively [4]. The $\psi(4160)$ resonance can decay into $\chi_{cJ}(1P)\gamma$ and $\chi_{cJ}(2P)\gamma$ via the radiative transitions.

Considering $\psi(4160)$ as a pure 2^3D_1 state, we predict

$$\Gamma[\psi(4160) \rightarrow \chi_{c0}(1P)\gamma] \simeq 21 \text{ keV}, \quad (90)$$

$$\Gamma[\psi(4160) \rightarrow \chi_{c1}(1P)\gamma] \simeq 6.5 \text{ keV}, \quad (91)$$

$$\Gamma[\psi(4160) \rightarrow \chi_{c2}(1P)\gamma] \simeq 8.8 \text{ keV}. \quad (92)$$

Our predictions of $\Gamma[\psi(4160) \rightarrow \chi_{c0,1}(1P)\gamma]$ are close to those of potential models [9], however, our prediction of $\Gamma[\psi(4160) \rightarrow \chi_{c2}(1P)\gamma]$ is about one order of magnitude larger than that of potential models [9] (see Tab. VI). It should be mentioned that the M2 transitions could give a 10–30% correction to the radiative decay widths by interfering with the E1 transitions. Combining the measured decay width of $\psi(4160)$ with our predicted partial widths, we estimate the branching fractions:

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c0}(1P)\gamma] \simeq 3.0 \times 10^{-4}, \quad (93)$$

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c1}(1P)\gamma] \simeq 1.0 \times 10^{-4}, \quad (94)$$

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c2}(1P)\gamma] \simeq 1.3 \times 10^{-4}. \quad (95)$$

The CLEO Collaboration only gave upper limits of $\mathcal{B}[\psi(4160) \rightarrow \chi_{c1}(1P)\gamma] < 7 \times 10^{-3}$ and $\mathcal{B}[\psi(4160) \rightarrow \chi_{c2}(1P)\gamma] < 13 \times 10^{-3}$ [77]. Our predictions are in the region of the measurements. Of course, we expect that more accurate observations can be carried out in future experiments.

Furthermore, we calculate the partial decay width of $\Gamma[\psi(4160) \rightarrow \chi_{c2}(2P)\gamma]$. Our result

$$\Gamma[\psi(4160) \rightarrow \chi_{c2}(2P)\gamma] \simeq 5.9 \text{ keV}, \quad (96)$$

is in agreement with the predictions in various potential models [9, 78]. At the same time, we can easily estimate the branching fraction

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c2}(2P)\gamma] \simeq 8.1 \times 10^{-5}. \quad (97)$$

The small branching fraction indicates that the $\chi_{c2}(2P)\gamma$ decay mode might be difficult to be observed in experiments.

The $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states are still not established, thus, we calculate the partial decay widths of the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ as functions of the masses of the $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states in a possible region. Our results are shown in Fig. 4. From the figure, it is seen that although the partial decay widths of $\Gamma[\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma]$ are sensitive to the mass of the final charmonium states, the partial decay widths are still fairly large when we take the upper limits for the masses of the final charmonium states. Thus, $\psi(4160)$ might be a good source to be used to look for the missing $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states via the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$, which is also suggested by Li, Meng and Chao in Ref. [78].

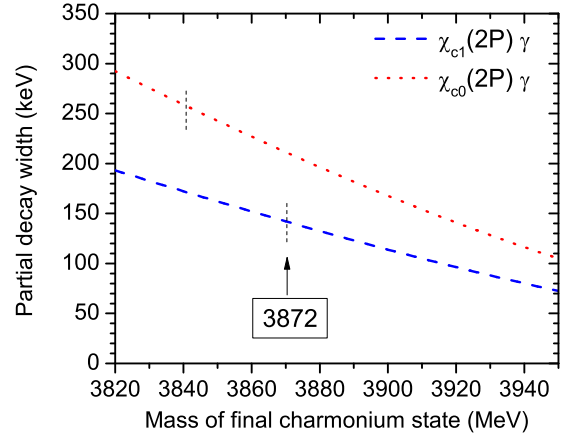


FIG. 4: (Color online) The partial decay widths of the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ as functions of the masses of the final states $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$.

Considering $X(3872)$ as the $\chi_{c1}(2P)$ state, we predict partial decay width and branching fraction of the process $\psi(4160) \rightarrow \chi_{c1}(2P)\gamma$:

$$\Gamma(\psi(4160) \rightarrow \chi_{c1}(2P)\gamma) \simeq 140 \text{ keV}, \quad (98)$$

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c1}(2P)\gamma] \simeq 2.0 \times 10^{-3}. \quad (99)$$

While if the mass of $\chi_{c0}(2P)$ is ~ 3840 MeV as predicted in Refs. [10, 70], the predicted partial decay width and branching fraction of the process $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma$ are

$$\Gamma(\psi(4160) \rightarrow \chi_{c0}(2P)\gamma) \simeq 259 \text{ keV}, \quad (100)$$

$$\mathcal{B}[\psi(4160) \rightarrow \chi_{c0}(2P)\gamma] \simeq 8.6 \times 10^{-3}. \quad (101)$$

According to our calculations, if $X(3872)$ is $\chi_{c1}(2P)$ and the mass of $\chi_{c0}(2P)$ is ~ 3840 MeV indeed, these states should be easily observed in the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ for their large branching fractions. To clarify the puzzles in $X(3872)$ and $\chi_{c0}(2P)$, we strongly suggest our experimental colleagues carry out observations of the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$ in the future.

2. $\psi_2(2D)$

The $\psi_2(2D)$ state is still not observed in experiments. Combined the measured mass of $\psi_1(2D)$ and the $2^3D_2 - 2^3D_1$ mass splitting from the quark model [9], the mass of $\psi_2(2D)$ may be about $M_{\psi_2(2D)} \simeq 4208$ MeV. The study of the radiative transitions $\psi_2(2D) \rightarrow \chi_{cJ}(1P, 2P)\gamma$ may be helpful to look for $\psi_2(2D)$ in experiments.

According to our calculations, we find that the partial widths of $\Gamma[\psi_2(2D) \rightarrow \chi_{c0}(1P, 2P)\gamma]$ are negligibly small. The $\chi_{c1}(1P)$, $\chi_{c2}(1P)$ and $\chi_{c2}(2P)$ states are well-established, thus, we can easily predict that

$$\Gamma[\psi_2(2D) \rightarrow \chi_{c1}(1P)\gamma] \simeq 26 \text{ keV}, \quad (102)$$

$$\Gamma[\psi_2(2D) \rightarrow \chi_{c2}(1P)\gamma] \simeq 10 \text{ keV}, \quad (103)$$

$$\Gamma[\psi_2(2D) \rightarrow \chi_{c2}(2P)\gamma] \simeq 64 \text{ keV}, \quad (104)$$

which are close to the predictions of NR potential model [9]. It should be mentioned that the E3 together with the M2 transitions have obvious corrections to the partial widths by interfering with E1 transitions (see Tab. VI).

The $\chi_{c1}(2P)$ state is still not established, thus, we calculate the partial decay width of the radiative transition $\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma$ as a function of the mass of the $\chi_{c1}(2P)$ state in a possible region. Our results are shown in Fig. 5. From the figure, it is seen that $\psi_2(2D)$ has a large transition rate into $\chi_{c1}(2P)\gamma$. The radiative partial width of $\Gamma[\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma]$ is about 100s keV, which is in agreement with the potential model predictions [9]. If $X(3872)$ is assigned as the $\chi_{c1}(2P)$ state, we predict that

$$\frac{\Gamma[\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma]}{\Gamma[\psi_2(2D) \rightarrow \chi_{c2}(2P)\gamma]} \simeq 5.6. \quad (105)$$

This ratio might be interesting to test the nature of $X(3872)$ in future experiments.

3. $\psi_3(2D)$

The $\psi_3(2D)$ state is still not established. Combined the measured mass of $\psi_1(2D)$ and the $2^3D_3 - 2^3D_1$ mass splitting from the quark model [9], the mass of $\psi_3(2D)$ is about

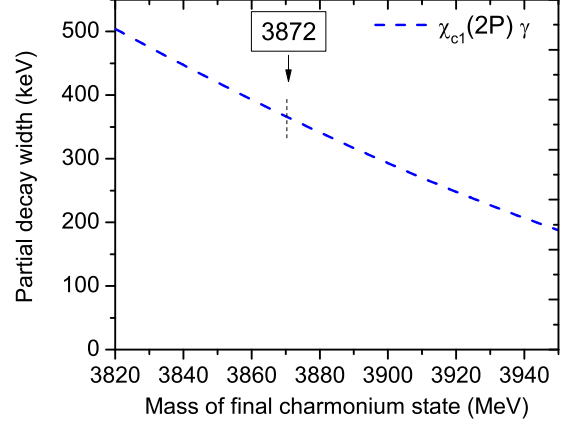


FIG. 5: (Color online) The partial decay widths of the radiative transitions $\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma$ as a function of the mass of the final state $\chi_{c1}(2P)$.

$M_{\psi_3(2D)} \simeq 4217$ MeV. The $\psi_3(2D)$ can decay into $\chi_{cJ}(1P)\gamma$ and $\chi_{cJ}(2P)\gamma$ via radiative transitions.

We find that $\chi_{c2}(1P)\gamma$ and $\chi_{c2}(2P)\gamma$ are the dominant radiative decay channels of $\psi_3(2D)$. We predict their partial decay widths:

$$\Gamma[\psi_3(2D) \rightarrow \chi_{c2}(1P)\gamma] \simeq 25 \text{ keV}, \quad (106)$$

$$\Gamma[\psi_3(2D) \rightarrow \chi_{c2}(2P)\gamma] \simeq 349 \text{ keV}, \quad (107)$$

which are compatible with the potential model results [9]. The large partial width of $\Gamma[\psi_3(2D) \rightarrow \chi_{c2}(2P)\gamma]$ indicates that $\chi_{c2}(2P)\gamma$ might be a good channel to find the missing state $\psi_3(2D)$ in future experiments.

Furthermore, in our calculations we find that the partial decay widths of $\Gamma[\psi_3(2D) \rightarrow \chi_{c0,c1}(2P)\gamma]$ are negligibly small, while the $\Gamma[\psi(2^3D_3) \rightarrow \chi_{c0,c1}(1P)\gamma]$ are sizeable. Our results are

$$\Gamma[\psi_3(2D) \rightarrow \chi_{c0}(1P)\gamma] \simeq 8.5 \text{ keV}, \quad (108)$$

$$\Gamma[\psi_3(2D) \rightarrow \chi_{c1}(1P)\gamma] \simeq 9.1 \text{ keV}. \quad (109)$$

It should be pointed out that in these two decay processes the E1 translations are nearly forbidden. Their partial widths are mainly contributed by the E3 translations.

4. $\eta_{c2}(2D)$

There is no information of $\eta_{c2}(2D)$ from experiments. Combined the measured mass of $\psi_1(2D)$ and the $2^1D_2 - 2^3D_1$ mass splitting from the quark model [9], the mass of $\eta_{c2}(2D)$ is $M_{\eta_{c2}(2D)} \simeq 4208$ MeV. We study the typical radiative transitions $\eta_{c2}(2D) \rightarrow h_c(1P, 2P)\gamma$. We find that $\eta_{c2}(2D)$ might have strong decay rates into both $h_c(1P)\gamma$ and $h_c(2P)\gamma$ channels. We predict that

$$\Gamma[\eta_{c2}(2D) \rightarrow h_c(1P)\gamma] \simeq 39 \text{ keV}, \quad (110)$$

$$\Gamma[\eta_{c2}(2D) \rightarrow h_c(2P)\gamma] \simeq 385 \text{ keV}, \quad (111)$$

where we take the mass of $h_c(2P)$ with $M_{h_c(2P)} = 3900$ MeV. Our predictions are in agreement with those from potential models [9] in magnitude (see Tab. VI). According to our analysis, the observations of the decay chain: $\eta_{c2}(2D) \rightarrow h_c(2P)\gamma$, $h_c(2P) \rightarrow \eta_c(1S)\gamma$ might be useful for the search for $\eta_{c2}(2D)$ and $h_c(2P)$ in experiments.

H. Radiative transitions of $3P$ states

Until now, no $3P$ charmonium states, $\chi_{c0,1,2}(3P)$ and $h_c(3P)$, have been established in experiments. Their predicted masses from various quark models have large uncertainties. The possible masses of the $\chi_{c0}(3P)$, $\chi_{c1}(3P)$, $\chi_{c2}(3P)$, and $h_c(3P)$ are in the ranges of (4.13 – 4.30) GeV, (4.17 – 4.38) GeV, (4.20 – 4.39) GeV, and (4.18 – 4.34) GeV, respectively [9–11]. Considering the uncertainties of their predicted masses, we plot the partial radiative decay width as a function of mass in Fig. 6. From the figure, one can see that the radiative decay properties of the $3P$ charmonium states are very sensitive to their masses. The EM decays of $\chi_{cJ}(3P)$ and $h_c(3P)$ are governed by the $\psi(3S)\gamma$ and $\eta(3S)\gamma$ channels, respectively. The partial widths of the $3P$ charmonium states decaying into the $2S$ states are also sizeable. If the $3P$ charmonium states have a larger mass, their partial width decaying into the $2D$ states will become comparable to those of into the $2S$ states.

It is interesting to note that the new $X(4350)$ state observed by the Belle Collaboration [80] was recommended as the $\chi_{c2}(3P)$ state by Liu *et al.* [69]. If $X(4350)$ corresponds to $\chi_{c2}(3P)$, its partial decay widths decaying into the $\psi(1S, 2S, 3S)\gamma$ channels are

$$\Gamma[X(4350) \rightarrow \psi(3S)\gamma] \simeq 341 \text{ keV}, \quad (112)$$

$$\Gamma[X(4350) \rightarrow \psi(2S)\gamma] \simeq 39 \text{ keV}, \quad (113)$$

$$\Gamma[X(4350) \rightarrow \psi(1S)\gamma] \simeq 8 \text{ keV}. \quad (114)$$

Combining these predicted partial widths with the measured width $\Gamma_{\text{exp}} \simeq 13$ MeV of $X(4350)$, we can estimate the branching fractions:

$$\mathcal{B}[X(4350) \rightarrow \psi(3S)\gamma] \simeq 2.6 \times 10^{-2}, \quad (115)$$

$$\mathcal{B}[X(4350) \rightarrow \psi(2S)\gamma] \simeq 2.2 \times 10^{-3}, \quad (116)$$

$$\mathcal{B}[X(4350) \rightarrow \psi(1S)\gamma] \simeq 6.2 \times 10^{-4}. \quad (117)$$

The large branching fractions of $\mathcal{B}[X(4350) \rightarrow \psi(2S, 3S)\gamma]$ indicate that $X(4350)$ should be observed in the $\psi(3S)\gamma$ and $\psi(2S)\gamma$ channels. Furthermore, as a candidate of $\chi_{c2}(3P)$ the radiative transition rate of $X(4350) \rightarrow \psi_3(2D)\gamma$ should be sizeable. Our predicted partial width and branching fraction are

$$\Gamma[X(4350) \rightarrow \psi_3(2D)\gamma] \simeq 31 \text{ keV}, \quad (118)$$

$$\mathcal{B}[X(4350) \rightarrow \psi_3(2D)\gamma] \simeq 2.4 \times 10^{-3}. \quad (119)$$

Thus, if the $X(4350)$ state is the $\chi_{c2}(3P)$ state indeed, $X(4350)$ might provide us a good source to look for the missing state $\psi(2^3D_3)$ via its radiative transition.

If $X(4350)$ is identified as the $\chi_{c2}(3P)$ state, its mass is very close to the predictions of GI potential model, thus, the masses

of $\chi_{c0}(3P)$, $\chi_{c1}(3P)$ and $h_c(3P)$ might be ~ 4.33 GeV [9]. To compare our predictions of the radiative transition properties of the $3P$ states with the potential models', in Tab. V we further list our results predicted with the masses from GI, NR [9], and SNR [10] potential models, respectively. It is found that the dominant decay channels predicted by us are consistent with those of potential models, although our predicted values show some obvious differences from those models'.

V. SUMMARY

In the constituent quark model framework, we systematically study the EM transitions of nS, nP ($n \leq 3$), and nD ($n \leq 2$) charmonium states. Without introducing any free parameters, we obtain a reasonable description of the EM transitions of the well-established low-lying charmonium states J/ψ , $\psi(2S)$, $\chi_{cJ}(1P)$, $h_c(1P)$ and $\psi(3770)$. It should be emphasized that the M2 transitions might give obvious corrections to some E1 dominant processes by interfering with the E1 transitions. We summarize our major results as follows.

For the radiative transition $J/\psi \rightarrow \eta_c(1S)\gamma$, there still exist puzzles in both theory and experiments. Our prediction of $\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma] \simeq 1.25$ keV is consistent with the average experimental data from the PDG [4], and the calculations with a relativistic quark model [24]. However, the calculations from potential models [9] and a recent lattice QCD approach [15] gave a large partial decay width, which is consistent with the recent measurements at KEDR $\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma] \simeq 2.98$ keV [54]. To clarify the discrepancies between different experiments and test various model predictions, more accurate observations of the $J/\psi \rightarrow \eta_c(1S)\gamma$ transition are needed in future experiments.

For the radiative transitions of $2S$ states, only some measurements for $\psi(2S)$ can be obtained until now. About the radiative transitions of $\eta_c(2S)$ it is found that our predictions have notable differences from the other predictions. While, our predictions about $\psi(2S)$ are comparable to the experimental observations and other model predictions. The partial width ratios for $\psi(2S)$, $\frac{\Gamma[\psi(2S) \rightarrow \chi_{c1,2}(1P)\gamma]}{\Gamma[\psi(2S) \rightarrow \chi_{c0}(1P)\gamma]}$ and $\frac{\Gamma[\psi(2S) \rightarrow \eta_c(1S)\gamma]}{\Gamma[J/\psi \rightarrow \eta_c(1S)\gamma]}$, have a strong model dependence. These partial width ratios for $\psi(2S)$, and the radiative transitions of $\eta_c(2S) \rightarrow h_c(1P)\gamma$, $J/\psi\gamma$ are worth to measuring to test various theoretical approaches.

For the radiative transitions of $1P$ charmonium states, our predictions are in reasonable agreement with the observations. There are large discrepancies between different model predictions of $\Gamma[h_c(1P) \rightarrow \eta_c\gamma]$, however, the present world data can not be used to test the various predictions for their large uncertainties. We hope more accurate measurements of the total decay widths of $\chi_{c2}(1P)$ and $h_c(1P)$, together with the observations of the $\chi_{c2}(1P) \rightarrow J/\psi\gamma$ and $h_c(1P) \rightarrow \eta_c\gamma$ processes can be carried out in future experiments.

For the radiative transitions of $1D$ charmonium states $\psi(3770)$ and $X(3823)$, our predictions are consistent with the observations. For the missing $1D$ charmonium states $\eta_{c2}(1D)$ and $\psi_3(1D)$, they have large radiative transition rates into $h_c(1P)\gamma$ and $\chi_{c2}(1P)\gamma$, respectively. Their corresponding par-

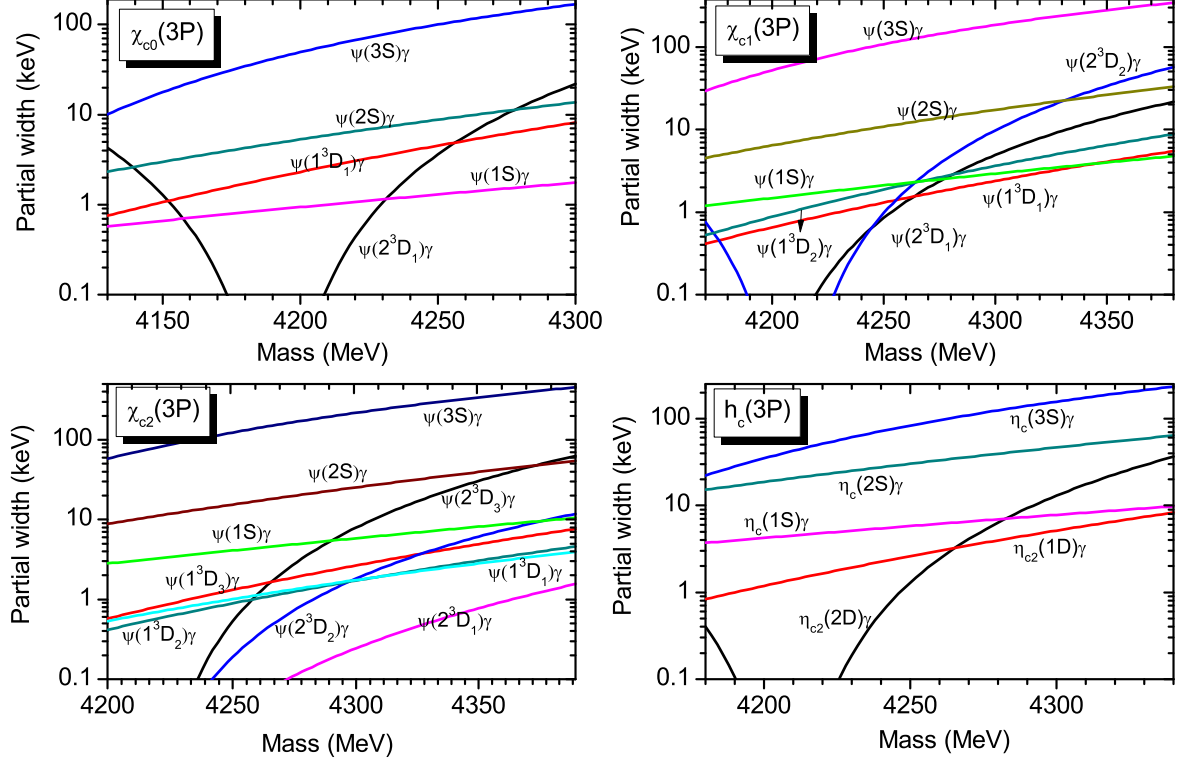


FIG. 6: (Color online) The partial radiative decay width (keV) of the radiative transitions of $3P$ states as a function of mass.

tial decay widths might be 200–300 keV, which indicates that the missing states $\eta_{c2}(1D)$ and $\psi_3(1D)$ are most likely to be established in the $\eta_{c2}(1D) \rightarrow h_c(1P)\gamma$ and $\psi_3(1D) \rightarrow \chi_{c2}(1P)\gamma$ processes, respectively.

The main radiative decay modes for the triplet $2P$ states $\chi_{cJ}(2P)$ are $\psi(2S)\gamma$ and $J/\psi(1S)\gamma$, while for the singlet $2P$ state $h_c(2P)$ are $\eta_c(1S, 2S)\gamma$. The partial decay widths of $\Gamma[\chi_{cJ}(2P) \rightarrow \psi(3770)\gamma]$ are also sizeable. Considering the $X(3872)$ resonance as the $\chi_{c1}(2P)$ state, we predict its radiative transition properties. Our predicted branching fraction ratio $R_{\psi'\gamma/\psi\gamma}$ supports $X(3872)$ as a candidate of $\chi_{c1}(2P)$. Further observations in the $\psi(3770)\gamma$ and $\psi_2(1D)\gamma$ channels are necessary to understand the nature of $X(3872)$. For the $\chi_{c0}(2P)$ state, we study its radiative transition properties by setting its mass with 3840 MeV and 3915 MeV, respectively. We find that the partial width ratio $\frac{\Gamma[\chi_{c0}(2P) \rightarrow \psi(2S)\gamma]}{\Gamma[\chi_{c0}(2P) \rightarrow J/\psi\gamma]}$ is sensitive to the mass of $\chi_{c0}(2P)$. Thus, we suggest the experimental Collaborations carry out some measurements of this ratio to establish the $\chi_{c0}(2P)$ state in experiments finally. For the missing state $h_c(2P)$, the predicted mass is $M_{h_c(2P)} \simeq 3900$ MeV. The partial decay widths into the $\eta_c(1S)\gamma$ and $\eta_c(2S)\gamma$ channels are rather sizeable. The discovery of the missing $h_c(2P)$ state in the $\eta_c(1S)\gamma$ and $\eta_c(2S)\gamma$ channels might be possible in future experiments.

For the radiative transitions of the $\psi(3S)$ state, i.e. $\psi(4040)$,

we predict that the partial decay widths of $\Gamma[\psi(3S) \rightarrow \chi_{cJ}(1P)\gamma]$ is only 2–3 keV. It might be a challenge to carry out accurate observations of the radiative transitions $\psi(4040) \rightarrow \chi_{cJ}(1P)\gamma$ in experiments for their small decay rates. However, the radiative transition rate of $\psi(4040) \rightarrow \chi_{cJ}(2P)\gamma$ is fairly large, which is about an order of magnitude larger than that of the $\psi(4040) \rightarrow \chi_{cJ}(1P)\gamma$. The missing states $\chi_{c1}(2P)$ and $\chi_{c0}(2P)$ may be produced through radiative decays of $\psi(4040)$, and be established in the $J/\psi\gamma$ and $\psi(2S)\gamma$ final states. Considering the $X(3940)$ as a candidate of the singlet $\eta_c(3S)$ state, we predict the radiative transition rates of $\eta_c(3S) \rightarrow h_c(1P, 2P)\gamma$ are small. Their branching ratios are about 10^{-5} , which indicates that if $X(3940)$ is the $\eta_c(3S)$ state, it might be difficult to be observed in its radiative transitions.

Taking the $\psi(4160)$ as a pure $\psi_1(2D)$ state, we calculate its radiative transition properties. We predict that the radiative transition rates of $\psi(4160)$ into the $2P$ states $\chi_{c0,1}(2P)$ are fairly large, the branching ratios of $\mathcal{B}[\psi(4160) \rightarrow \chi_{c0,1}(2P)\gamma]$ are about $10^{-2} - 10^{-3}$. Thus, $\psi(4160)$ might be a very good source to be used to look for the missing $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ states via the radiative transitions $\psi(4160) \rightarrow \chi_{c0}(2P)\gamma, \chi_{c1}(2P)\gamma$. For the missing $2D$ states $\psi_2(2D)$, $\psi_3(2D)$ and $\eta_{c2}(2D)$, we find that their partial widths of $\Gamma[\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma]$, $\Gamma[\psi_3(2D) \rightarrow \chi_{c2}(2P)\gamma]$, and $\Gamma[\eta_{c2}(2D) \rightarrow h_c(2P)\gamma]$ are relatively large (about 100s

keV). It is possible to establish the missing states $\psi_2(2D)$, $\psi_3(2D)$ and $\eta_{c2}(2D)$ in the radiative decay chains $\psi_2(2D) \rightarrow \chi_{c1}(2P)\gamma \rightarrow \psi(2S)\gamma\gamma$, $\psi_3(2D) \rightarrow \chi_{c2}(2P)\gamma \rightarrow \psi(2S)\gamma\gamma$, and $\eta_c(2^1D_2) \rightarrow h_c(2P)\gamma \rightarrow \eta_c(1S)\gamma\gamma$, respectively.

The radiative decay properties of the $3P$ charmonium states are very sensitive to their masses. The EM decays of $\chi_{cJ}(3P)$ and $h_c(3P)$ are governed by the $\psi(3S)\gamma$ and $\eta(3S)\gamma$ channels, respectively. The partial widths of the $3P$ charmonium states decaying into the $2S$ states are also sizeable. The partial decay widths for the transitions $\chi_{cJ}(3P) \rightarrow \psi(3S)\gamma$ and $h_c(3P) \rightarrow \eta(3S)\gamma$ are about $10s - 100s$ of keV. Thus, the observations of the missing $3P$ charmonium states in the $\psi(3S)\gamma$ and $\eta(3S)\gamma$ channels are necessary in experiments. Furthermore, taking the newly observed resonance $X(4350)$ as $\chi_{c2}(3P)$ as suggested by Liu *et al.* [69], we find that the branching ratios of $\mathcal{B}[X(4350) \rightarrow \psi(3S)\gamma]$ and $\mathcal{B}[X(4350) \rightarrow \psi(2S)\gamma]$ are $\sim 10^{-2}$ and $\sim 10^{-3}$, respectively. The $X(4350)$ resonance is most likely to be observed in the $\psi(2S, 3S)\gamma$ channels. We also find $X(4350)$ has a large branching ratio $\mathcal{B}[X(4350) \rightarrow \psi_3(2D)\gamma] \simeq 2.4 \times 10^{-3}$, thus, $X(4350)$ might provide us a good source to look for the missing $\psi_3(2D)$ via

its radiative transition.

Finally, it should be pointed out that the harmonic oscillator wave functions used by us might bring some uncertainties to our predictions. Furthermore, the couple-channel effects (i.e., the intermediate hadronic meson loops) might provide important corrections to the EM decay properties of the charmonium states lying above the open charm thresholds [57], which might affect our conclusions of the higher charmonium states. Our quark model approach is also used to study the EM decay properties of bottomonium states. For clarity, our results about the bottomonium states will be reported in another work.

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TABLE III: Partial widths (keV) of the M1 radiative transitions for some low-lying S -wave charmonium states.

Initial state	Final state	E_γ (MeV)				Γ_{M1} (keV)				Γ (keV) Exp.
		Ref.[24]	NR[9]	GI [9]	Ours	Ref.[24]	NR	GI	Ours	
J/ψ	$\eta_c(1S)$	115	116	115	111	1.05	2.9	2.4	1.25	1.58 ± 0.37
$\psi(2S)$	$\eta_c(2S)$	32	48	48	47	0.043	0.21	0.17	0.10	0.21 ± 0.15
	$\eta_c(1S)$	639	639	638	635	0.95	4.6	9.6	2.41	1.24 ± 0.29
$\eta_c(2S)$	J/ψ	514	501	501	502	1.53	7.9	5.6	1.64	
$\psi(3S)$	$\eta_c(3S)$		29	35	99		0.046	0.067	0.878	
	$\eta_c(2S)$		382	436	381		0.61	2.6	0.279	
	$\eta_c(1S)$		922	967	918		3.5	9.0	0.445	

TABLE IV: Partial widths (keV) of the radiative transitions between the established charmonium states. For comparison, the predictions from the relativistic quark model [24], NR and GI models [9] and SNR model [10] are listed in the table as well. The experimental average data are taken from the PDG. Γ_{E1} and Γ_{EM} stands for the E1 and EM transition widths, respectively.

Initial state	Final state	E_γ (MeV)				Γ_{E1} (keV)				Γ_{EM} (keV)	
		Ref. [24]	NR/GI [9]	SNR [10]	Ours	Ref.[24]	NR/GI [9]	SNR _{0/1} [10]	Ours	Ours	Exp.
$\psi(2S)$	$\chi_{c2}(1P)$	128	128 / 128	128	128	18.2	38 / 24	43/34	15	14	25.2 ± 2.9
	$\chi_{c1}(1P)$	171	171 / 171	171	171	22.9	54 / 29	62/36	20	22	25.5 ± 2.8
	$\chi_{c0}(1P)$	259	261 / 261	261	261	26.3	63 / 26	74/25	22	26	26.3 ± 2.6
$\chi_{c2}(1P)$	J/ψ	430	429 / 429	429	429	327	424 / 313	473/309	404	467	371 ± 34
$\chi_{c1}(1P)$		389	390 / 389	390	390	265	314 / 239	354/244	313	275	285 ± 14
$\chi_{c0}(1P)$		305	303 / 303	303	303	121	152 / 114	167/117	159	128	133 ± 8
$h_c(1P)$	$\eta_c(1S)$	504	504 / 496	504	499	560	498 / 352	764/323	587	587	357 ± 280
$\psi_1(1D)$	$\chi_{c2}(1P)$	234	208/208	213	215	6.9	4.9/3.3	5.8/4.6	3.4	2.6	< 17.4
	$\chi_{c1}(1P)$	277	250/251	255	258	135	125/77	150/93	83	70	81 ± 27
	$\chi_{c0}(1P)$	361	338/338	343	346	355	403/213	486/197	245	218	202 ± 42
$\psi_2(1D)$	$\chi_{c2}(1P)$	248	236/272	234	258	59	64/66	70/55	50	42	
	$\chi_{c1}(1P)$	291	278/314	276	299	215	307/268	342/208	226	227	
$\psi_1(2D)$	$\chi_{c2}(1P)$		559/590		587		0.79/0.027		0.46	8.8	
	$\chi_{c1}(1P)$		598/628		625		14 / 3.4		10	6.5	
	$\chi_{c0}(1P)$		677/707		704		27 / 35		27	21	
	$\chi_{c2}(2P)$		183/210		256		5.9/ 6.3		7.2	5.9	
$\eta_c(2S)$	$h_c(1P)$	128	111 / 119	109	112	41	49 / 36	146/104	18	18	
$\psi(3S)$	$\chi_{c2}(2P)$		67 / 119	119	111		14 / 48		20	19	
	$\chi_{c2}(1P)$		455 / 508	508	455		0.70 / 13		2.6	2.2	
	$\chi_{c1}(1P)$		494 / 547	547	494		0.53 / 0.85		2.6	3.0	
	$\chi_{c0}(1P)$		577 / 628	628	577		0.27 / 0.63		2.2	3.1	
$\chi_{c2}(2P)$	$\psi_2(1D)$		168 / 139	139	103		17 / 5.6		1.4	1.5	
	$\psi_1(1D)$		197 / 204	204	146		1.9 / 1.0		0.26	0.32	
	$\psi(2S)$		276 / 282	235	234		304 / 207	225/100	123	133	
	J/ψ		779 / 784	744	742		81 / 53	101/109	26	34	

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TABLE V: Partial widths (keV) for the radiative transitions whose initial and/or final charmonium states have not been established. In the calculations, the masses of these well-established states are taken the average values from the PDG. While, the masses of the missing charmonium states are adopted the NR, GI and SNR potential model predictions, which correspond to our three quark model predictions QM₁, QM₂ and QM₃, respectively, for each radiative transition. For comparison, the predictions from the NR and GI models [9] and SNR model [10] are listed in the table as well.

Initial state	Final state	E_γ (MeV)			Γ_{E1} (keV)					Γ_{EM} (keV)		
		NR/GI [9]	SNR [10]	QM _{1/2/3}	NR/GI [9]	SNR _{0/1} [10]	QM ₁	QM ₂	QM ₃	QM ₁	QM ₂	QM ₃
$\psi(3S)$	$\chi_{c1}(2P)$	113/145		113/86/137	39/43		12	5.5	21	13	5.7	22
	$\chi_{c0}(2P)$	184/180		184/122/193	54/22		16	5.1	19	18	5.5	21
	$\eta_c(3S)$	108/108		108/107/82	105/64		32	31	14	32	31	14
	$h_c(1P)$	485/511		485/503/439	9.1/28		6.9	8.7	3.6	6.9	8.7	3.6
$\chi_{c2}(2P)$	$\psi_3(1D)$	163/128		119/77/126	88/29		12.0	3.4	14.1	11.7	3.3	13.8
$\chi_{c1}(2P)$	$\psi_2(1D)$	123/113		101/128/77	35/18		6.6	13	3.0	6.6	13	3.0
	$\psi_1(1D)$	152/179		144/171/121	22/21		6.2	10	3.7	6.8	11	4.1
	$\psi(2S)$	232/258	182	232/258/209	183/183	103/60	120	161	90	111	148	84
	J/ψ	741/763	697	741/763/721	71/14	83/45	26	31	22	20	24	17
$\chi_{c0}(2P)$	$\psi_1(1D)$	81/143		73/136/63	13/51		3.4	21	2.2	3.5	22	2.3
	$\psi(2S)$	162/223	152	257/223/248	64/135	61/44	159	108	144	133	93	121
	J/ψ	681/733	672	681/733/673	56/1.3	74/9.3	16	24	15	9.4	14	8.8
	$h_c(2P)$	133/117		133/117/110	60/27		20	14	11	20	14	11
$h_c(2P)$	$\eta_{c2}(1D)$	285/305	261	284/304/260	280/218	309/108	208	249	164	208	249	164
	$\eta_c(2S)$	839/856	818	835/853/815	140/85	134/250	51	58	45	51	58	45
	$\eta_c(1S)$											
	$\eta_c(1S)$											
$\chi_{c2}(3P)$	$\psi_3(2D)$	147/118		147/118/104	148/51		44	23	16	43	23	16
	$\psi_2(2D)$	156/127		156/127/107	31/10		9.2	5.1	3.1	10	5.6	3.3
	$\psi_1(2D)$	155/141		124/144/17	2.1/0.77		0.32	0.48	0	0.38	0.60	0
	$\psi_3(1D)$	481/461		481/461/389	0.049/6.8		4.8	3.6	1.3	5.5	4.2	1.5
	$\psi_2(1D)$	486/470		466/484/367	0.01/0.13		0.70	0.88	0.15	2.1	2.7	0.47
	$\psi_1(1D)$	512/530		505/523/408	0.00/0.00		0.08	0.09	0.02	1.9	2.3	0.54
	$\psi(3S)$	268/231		268/287/165	509/199		235	278	63	257	306	66
	$\psi(2S)$	585/602		585/602/490	55/30		24	29	8.2	30	35	9.7
	J/ψ	1048/1063		1048/1063/964	34/19		4.6	5.2	2.1	6.5	7.5	3.0
	$\psi_2(2D)$	112/108		112/108/77	58/35		18	16	6.0	18	16	6.0
	$\psi_1(2D)$	111/121		79/124/0	19/15		2.2	8.0	0	2.3	8.7	0
	$\psi_2(1D)$	445/452		425/466/340	0.035/4.6		2.0	3.5	0.47	2.5	4.4	0.60
$\chi_{c1}(3P)$	$\psi_1(1D)$	472/512		465/505/381	0.014/0.39		1.1	1.9	0.32	1.5	2.6	0.41
	$\psi(3S)$	225/212		225/268/136	303/181		148	235	36	137	215	35
	$\psi(2S)$	545/585		545/585/463	45/8.9		16	24	5.8	13	20	5.0
	J/ψ	1013/1048		1013/1048/941	31/2.2		3.4	4.6	1.7	2.4	3.2	1.3
	$\psi_1(2D)$	43/97		11/100/0	4.4/35		0.02	17	0	0.02	18	0
	$\psi_1(1D)$	410/490		403/483/338	0.037/9.7		1.9	5.8	0.60	2.1	6.8	0.66
	$\psi(3S)$	159/188		159/245/90	109/145		57	185	11	51	156	10
	$\psi(2S)$	484/563		484/563/421	32/0.045		7.7	19	3.2	5.4	13	2.3
	J/ψ	960/1029		960/1029/905	27/1.5		2.1	3.9	1.2	0.95	1.7	0.58
	$h_c(3P)$	119/109		119/109/84	99/48		28	22	10	28	22	10
	$\eta_{c2}(2D)$	453/454		453/454/370	0.16/5.7		3.9	4.0	1.1	6.3	6.4	1.8
	$\eta_{c2}(1D)$	229/246		229/247/189	276/208		156	189	92	156	189	92
$h_c(3P)$	$\eta_c(3S)$	593/627		592/626/510	75/43		39	54	16	39	54	16
	$\eta_c(2S)$	1103/1131		1099/1128/1028	72/38		6.9	8.6	3.8	6.9	8.6	3.8
	$\eta_c(1S)$											
	$\eta_c(1S)$											

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TABLE VI: Partial widths (keV) for the radiative transitions of unestablished D -wave charmonium states. In the calculations, the masses of these well-established states are taken the average values from the PDG. While, the masses of the missing charmonium states are adopted the NR, GI and SNR potential model predictions, which correspond to our three quark model predictions QM₁, QM₂ and QM₃, respectively, for each radiative transition. For comparison, the predictions from the relativistic quark model [24], NR and GI models [9], and SNR model [10] are listed in the table as well.

Initial state	Final state	E_γ (MeV)				Γ_{E1} (keV)						Γ_{EM} (keV)		
		[24]	NR/GI [9]	SNR [10]	QM _{1/2/3}	[24]	NR/GI [9]	SNR _{0/1} [10]	QM ₁	QM ₂	QM ₃	QM ₁	QM ₂	QM ₃
$\psi_3(1D)$	$\chi_{c2}(1P)$	250	242/282	236	242/282/235	156	272/296	284/223	167	256	155	175	271	162
	$\chi_{c1}(1P)$				284/323/277				0	0	0	0.54	1.1	0.48
	$\chi_{c0}(1P)$				371/410/365				0	0	0	0.60	1.2	0.53
$\eta_{c2}(1D)$	$h_c(1P)$	275	264/307	260	264/299/261	245	339/344	575/375	214	302	208	214	302	208
$\psi_3(2D)$	$\chi_{c2}(1P)$		566/609		566/609/511		29/16		13	21	7.0	16	25	8.5
	$\chi_{c1}(1P)$				604/647/549				0	0	0	6.2	9.1	3.6
	$\chi_{c0}(1P)$				684/726/630				0	0	0	6.2	8.5	4.0
$\psi_2(2D)$	$\chi_{c2}(1P)$		558/602		558/601/508		7.1/0.62		3.1	4.8	1.7	6.7	10	3.8
	$\chi_{c1}(1P)$		597/640		597/639/547		26/23		14	21	8.0	17	26	10
$\eta_{c2}(2D)$	$h_c(1P)$		585/634		585/628/534		40/25		16	25	9.2	26	39	15
$\psi_3(2D)$	$\chi_{c2}(2P)$		190/231		233/280/172		239/272		202	330	87	212	349	90
	$\chi_{c1}(2P)$				235/256/197				0.24	0.42	0.07	0.44	0.72	0.16
	$\chi_{c0}(2P)$				303/290/253				0.65	0.49	0.19	0.70	0.52	0.20
$\psi_2(2D)$	$\chi_{c2}(2P)$		182/223		225/272/169		52/65		46	76	21	39	64	19
	$\chi_{c1}(2P)$		226/247		226/247/194		298/225		140	178	92	140	178	92
$\psi_1(2D)$	$\chi_{c1}(2P)$		227/234		258/231/280		168/114		110	82	137	92	70	113
	$\chi_{c0}(2P)$		296/269		325/266/334		483/191		268	162	287	240	146	256
$\eta_{c2}(2D)$	$h_c(2P)$		218/244		218/244/187		336/296		168	230	109	169	231	109

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